Algorithmic and Economic Aspects of Networks

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Learning in Networks

Last lecture:

- Actions chosen *probabilistically*
- Payoffs action-dependent and unknown

Trick was to learn to play a high-payoff action.

Game Theory in Networks

This lecture:

- Actions chosen *strategically*
- Payoffs depend on the set of people that choose each action

Trick is to strategize based on others' actions.

Game Theory in Networks

Example: Should athletes dope?

+ improves performance (esp. if competitors dope)
- penalities if caught

Beneficial to dope if enough competitors dope.

Game Theory in Networks

Example: Should you install (unsecured) wireless internet access?

- costs money

+ you can check email all night long

Beneficial to buy if neighbors don't.

Game Theory

Model actions and payoffs as a game with:

a set of players {1, …, n}

an actions space S_i for each player i

a payoff $u_i(s)$ to each player i for action profile s in $S_1 \times \dots \times S_n$

Let’s Play a Game

The Median Game

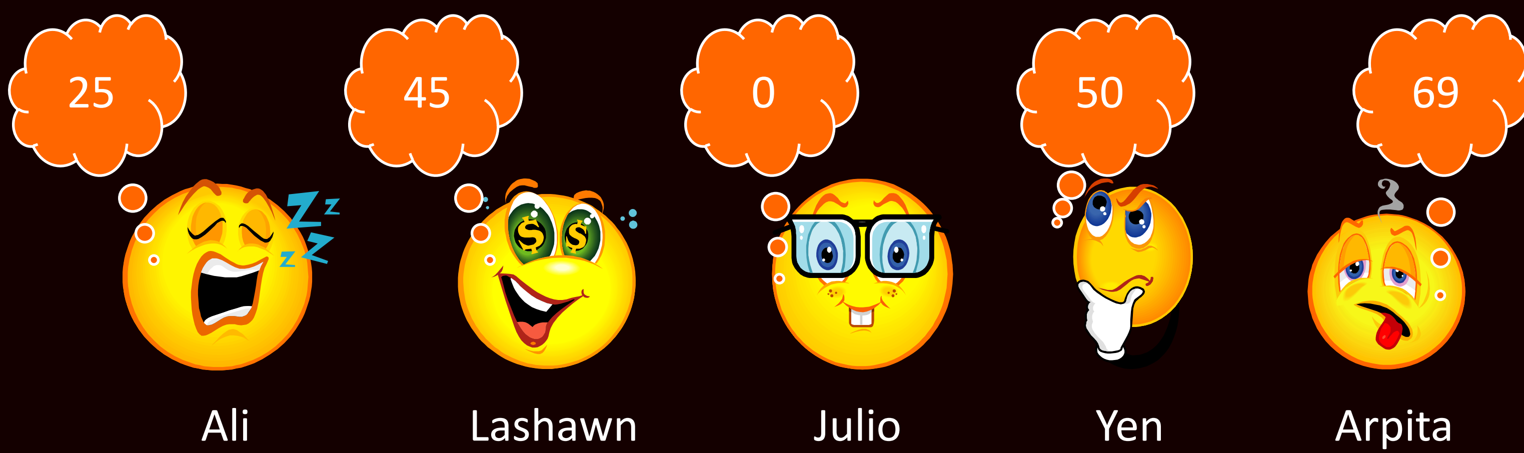
players = you

actions = {1, …, 100}

$u_i(s)$ = 1 if s_i is closest to 2/3 of median, 0 otherwise (ties broken randomly)

The Median Game

Example: If the numbers are



Median is 45, and Ali wins because his guess is closest to 2/3 of the median, or 30.

The Median Game

Arun:	32
Ted:	40
Matt:	20
Eric:	20
Michael:	49
Trevor:	19
Bach:	35
Mykell:	22
William:	10
Patrick:	35
Jia:	44

Bi-Matrix Games

Two players, Row and Column

- Row has m strategies
- Column has n strategies

Bi-Matrix Games

Payoffs represented by an (m x n) matrix A whose entries are pairs of numbers (x, y)

A_{ij} = (x, y) means Row earns x and Column earns y when Row plays i and Column plays j

Bi-Matrix Games

Example: Prisoners' Dilemma

	Deny	Confess
Deny	(-1, -1)	(-3, 0)
Confess	(0, -3)	(-2, -2)

Game Theory

Given a game, can we predict
which strategies the players will play?

Predicting Game Play

Example: Prisoners' Dilemma

	Deny	Confess
Deny	(-1, -1)	(-3, 0)
Confess	(0, -3)	(-2, -2)

Dominant Strategies

In Prisoner's Dilemma, best strategy is to confess *no matter what the other player does*

This is a dominant strategy equilibrium.

(there is a single best response to all possible sets of actions of your opponent(s))

Dominant Strategies

Dominant strategy equilibria don't always exist.

Median Game:

- if everyone chooses 90, best choice = 60
- if everyone chooses 60, best choice = 40

Pure Nash Equilibria

Q. How should one play the median game?

A. Only strategy profile in which everyone is playing a best response is the all-ones profile.

This is a pure Nash equilibrium.

(everyone simultaneously plays a best response to actions of opponent(s))

Pure Nash Equilibria

Pure Nash equilibria aren't always unique.

Example: Coordination game

	Theater	Football
Theater	(5, 4)	(2, 1)
Football	(1, 2)	(4, 5)

Pure Nash Equilibria

Pure Nash equilibria don't always exist.

Example: Matching pennies game

	Heads	Tails
Heads	(1, -1)	(-1, 1)
Tails	(-1, 1)	(1, -1)

Mixed Nash Equilibria

Let players chose strategies probabilisitically.

		1/2 Heads	1/2 Tails
1/2	Heads	(1, -1)	(-1, 1)
1/2	Tails	(-1, 1)	(1, -1)

Expected Payoff: (1/4) (1 + -1 + -1 + 1) = 0

Mixed Nash Equilibria

This is the maximum payoff Row can acheive fixing the strategy of Column

	1/2	1/2
p	(1, -1)	(-1, 1)
1-p	(-1, 1)	(1, -1)

$$E[\rho_{Row}] = (1/2)p - (1/2)(1-p) - (1/2)(p) + (1/2)(1-p) = 0$$

Mixed Nash Equilibria

Always exist (Nash 1950), but …

a game may have multiple NE

it may be hard to compute even one

Recap

Equilibrium notions:

dominant strategy << pure NE << mixed NE

may not exist | always exist
computable (if exist) | maybe not computable

unique | maybe not unique

Graphical Games

Defn. A graphical game is a normal form game in which the payoff to i depends only on her neighbors in the graph G.

Graphical Games

Median Game: complete graph

Doping Game: (i,j) are neighbors if they are in the same competition

Wireless Internet Game: (i,j) are neighbors if they can get each others' wireless signals

Graphical Games

For purpose of lecture,

we will assume two actions labeled 0 and 1

we will assume undirected graphs

Let $u_i(x_i, x_{N(i)})$ be payoff to i when i plays x_i and neighbors N(i) play according to profile $x_{N(i)}$

Games of Complements

Benefit of action increases as more neighbors perform action, e.g., doping game.

$$u_i(1, x_{N(i)}) > u_i(0, x_{N(i)})$$

if and only if

# of j in N(i) taking action 1 is > T_i

Threshold

Games of Substitutes

Benefit of action decreases as more neighbors perform action, e.g., wireless game.

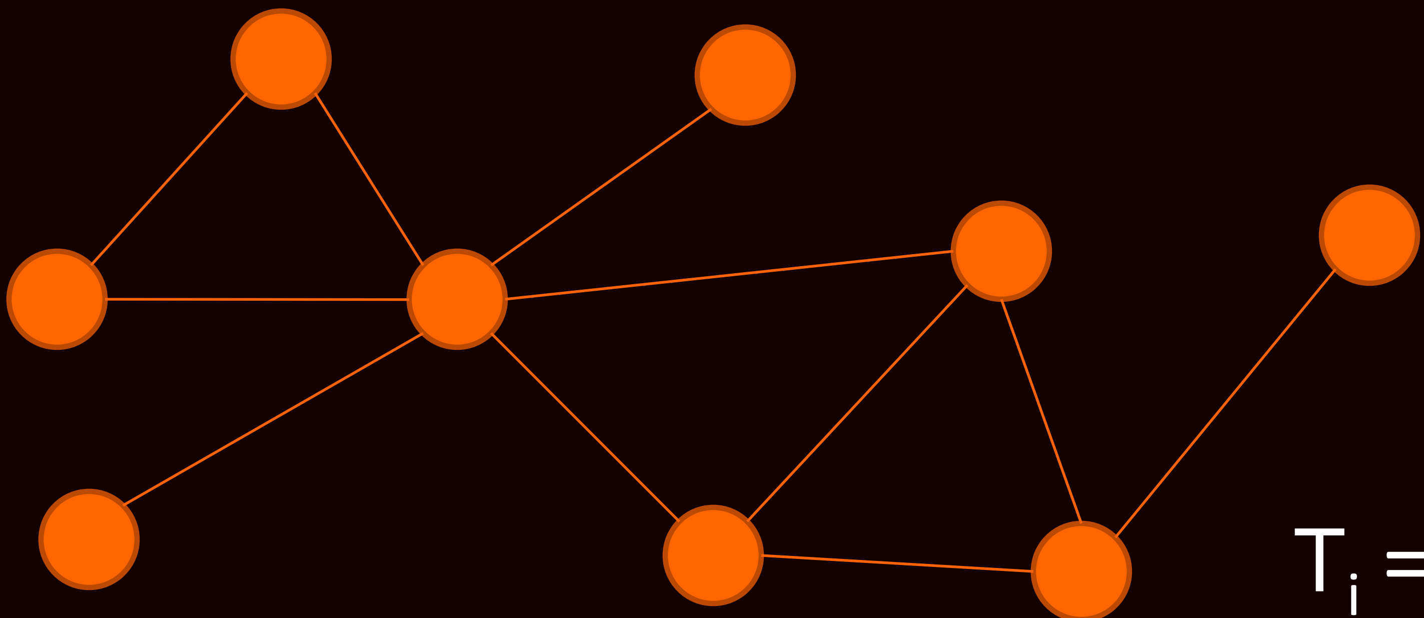
$$u_i(1, x_{N(i)}) > u_i(0, x_{N(i)})$$

if and only if

# of j in N(i) taking action 1 is $< T_i$

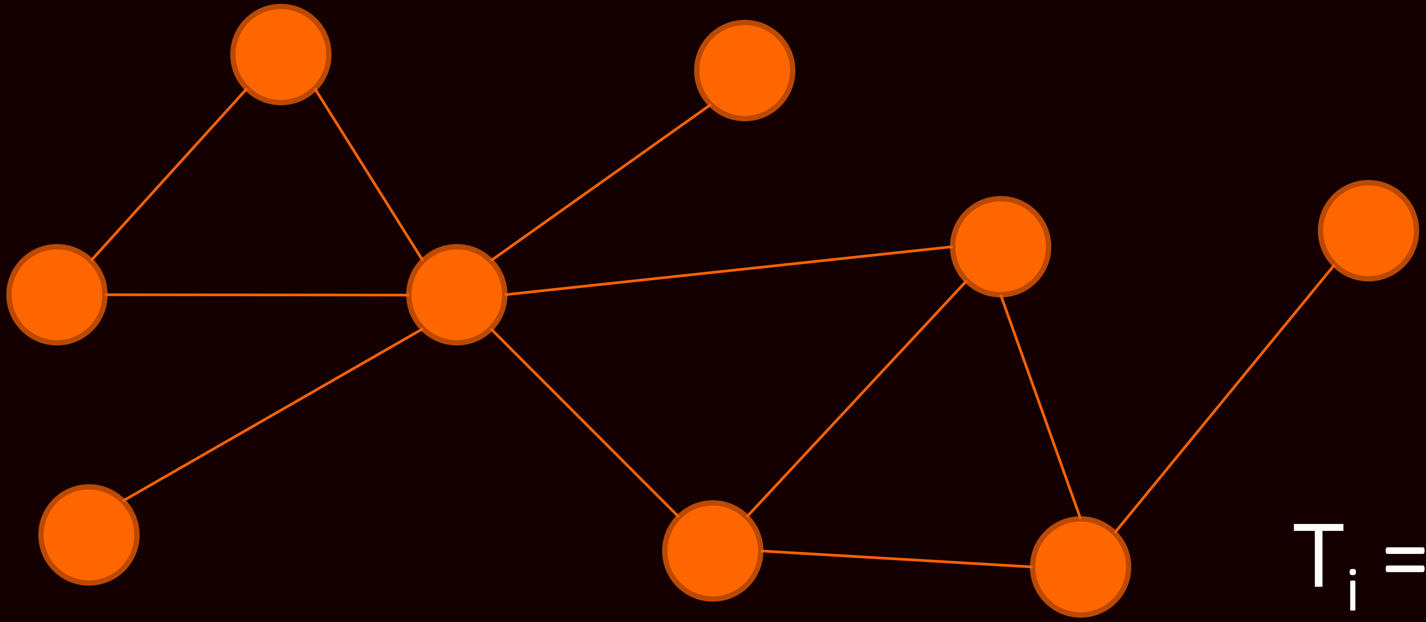
Threshold

Equilibria: Complements



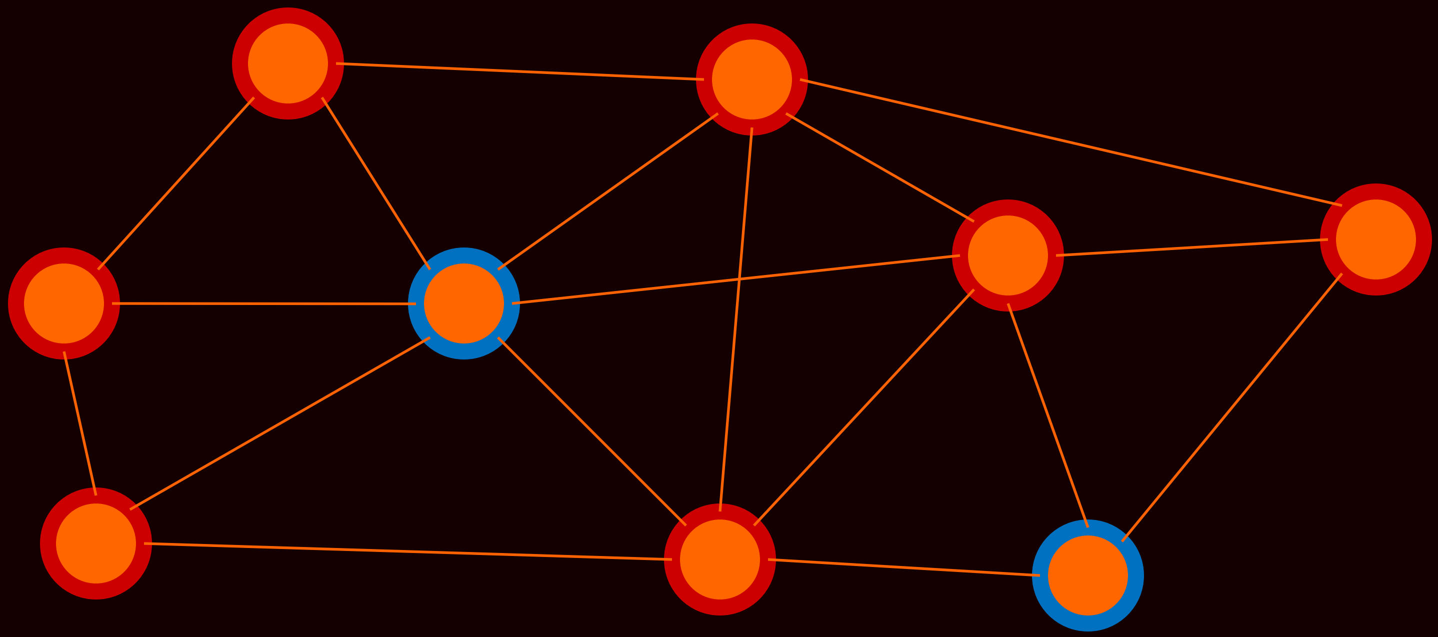
$T_i = 2$

Equilibria: Substitutes



$T_i = 1$

Smoking Game



Conformists: Smoke if ≥ 1/2 neighbors smoke.

Rebels: Smoke if no neighbor smokes.

Questions

How does network structure effect equilibria?

How can one design the network to produce optimal equilibria?

Dynamic Behavior

Start from an initial configuration and let players update strategies over time

what equilibrium results?

how's it depend on initial configuration?

how's it depend on network structure?

Dynamic Behavior

Assume players act
myopically and sequentially.

Product Adoption Model

Having similar behaviors/technologies as neighbors facilitates interaction (improves communication, understanding, etc.)

Given initial adoption, can we “buy off” some customers to get everyone to use another product?

Diffusion of Innovation

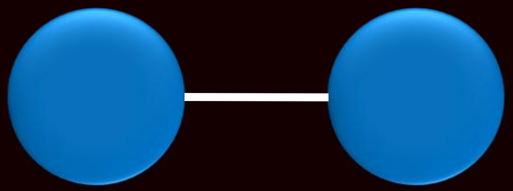
1. Each person can only adopt one behavior.

2. You gain more if you have the same behavior as your peers.

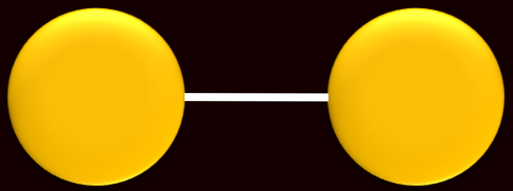
3. As people update behaviors to improve gains, diffusion happens.

Two Nodes

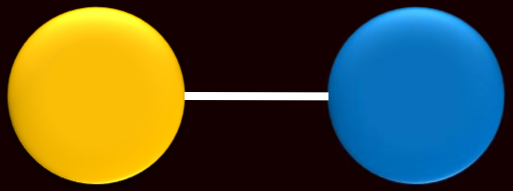
If both adopt A, get satisfaction a from coordination.



If both adopt B, get satisfaction b from coordination.



Adopt different behaviors, no coordination, zero satisfaction.



Many Nodes

Node communicates using same behavior with each of its neighbors

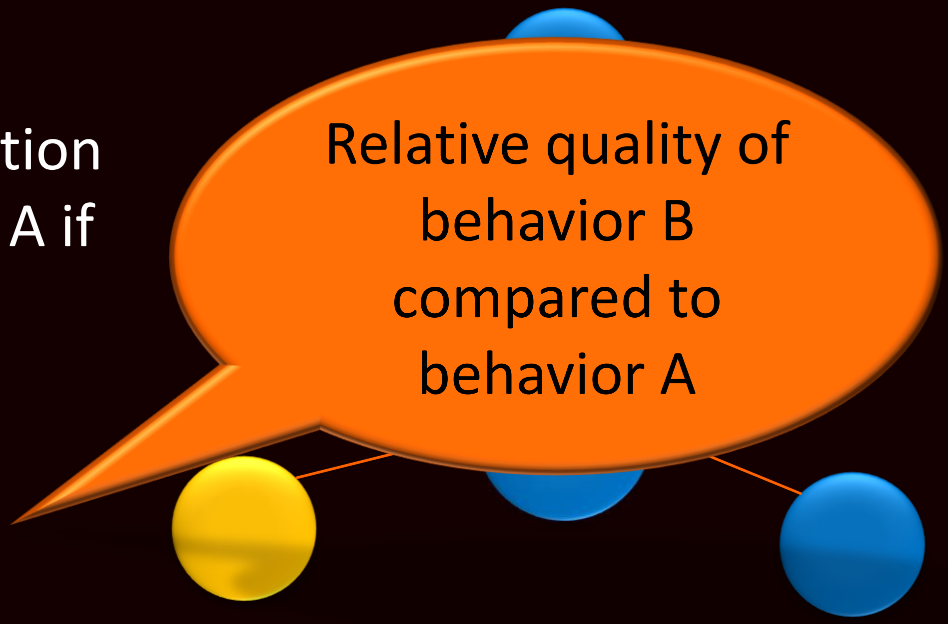
Total satisfaction is sum of edge satisfactions

Suppose node v has d neighbors, of which fraction p use A. Then v will use A if

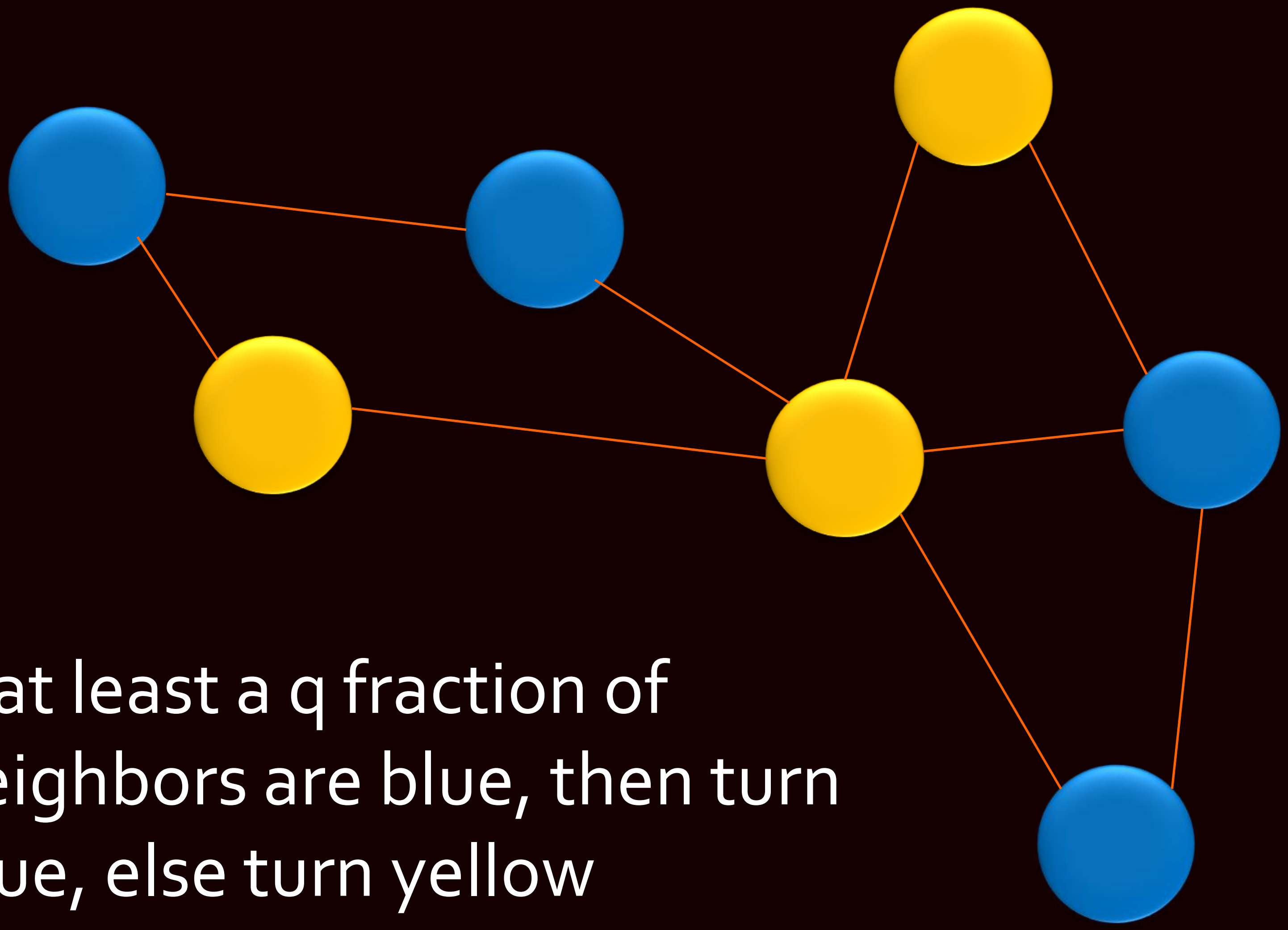
$$pda > (1-p)db$$

or

$$p > b / (a+b) = q$$



Choosing Behaviors



If at least a q fraction of neighbors are blue, then turn blue, else turn yellow

Coordination Game, cont'd

Player 1/ Player 2	A	B
A	(1-q,1-q)	(0,0)
B	(0,0)	(q,q)

Payoff Matrix

Payoff of a node is the sum over all incident edges.

An equilibrium is a strategy profile where no player can gain by changing strategies.

Diffusion Process

Some nodes are endowed with a fixed strategy

Remaining nodes move sequentially in an arbitrary order infinitely often

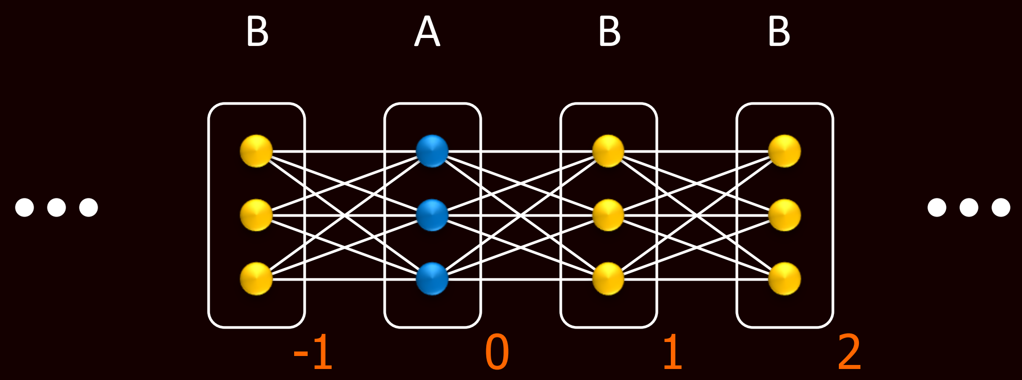
When asked to move, a node myopically chooses behavior that maximizes payoff

“If > q fraction of neighbors play A, then play A.”

Diffusion Questions

- A game-theoretic model of diffusion
  - Question: can a new behavior spread through a network where almost everyone is initially using another behavior?
- Can compatibility help?

Basic Diffusion Example 1

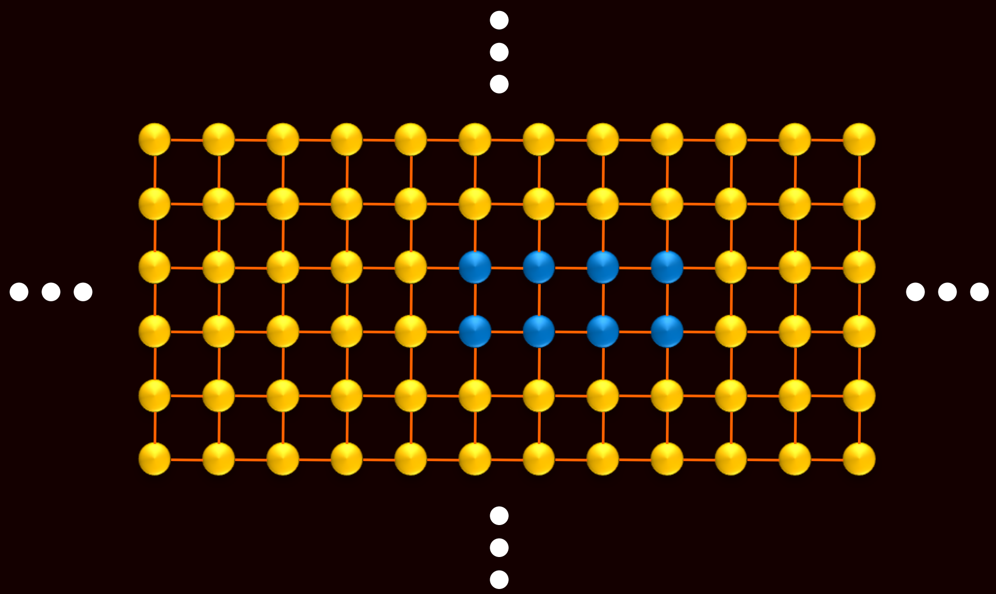


Endow group 0 with blue strategy

``If at least a q fraction of neighbors use blue strategy, then use blue strategy."

If $q < ½$, whole graph will turn blue

Basic Diffusion Example 2



Endow any group with blue

``If at least a q fraction of neighbors use blue, then use blue.”

Need $q < 1/4$ for behavior to spread

Contagion

Let G be a Δ-regular infinite graph

Starting from an all-B equilibrium, endow a finite set S of nodes (the "early adopters") with behavior A

A contagion results if myopic best-response moves cause all nodes to use A eventually

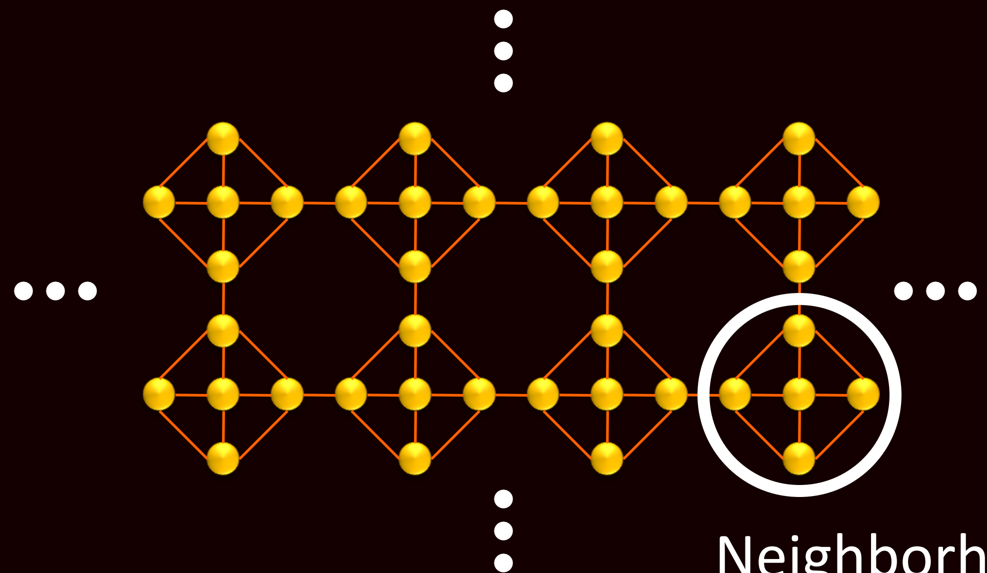
Contagion

Higher q makes contagion harder

Max q for which contagion happens for some finite set of nodes is the contagion threshold

Thm. [Morris, 2000]: For every graph G, the contagion threshold is at most ½.

What Stops Contagion?



A neighborhood with cohesion p(S) is a set S of nodes such that each node has at least a p fraction of its neighbors in S

Contagion

If there exists an infinite neighborhood S with $p(S) > 1 - q$, then contagion can't "break in"

If $p(S) < 1 - q$ for every infinite neighborhood S, then contagion happens

Thm. [Morris, 2000]: The contagion threshold of a graph is the largest q such that $q < 1 - p(S)$ for all infinite neighborhoods S.

Can compatibility help?

Compatibility

Coexistence of multiple behaviors or technologies, with varying degrees of compatibility

Examples:

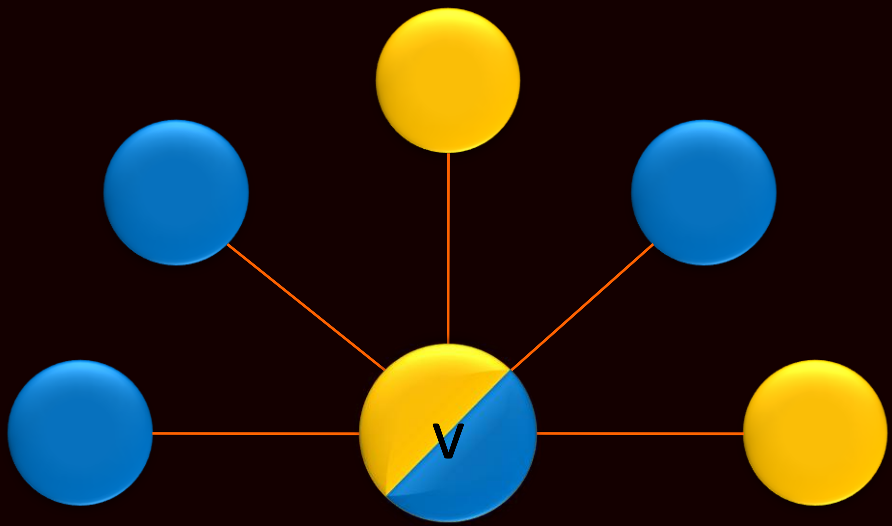
- Human languages: multi-lingual people
- Cell phone companies: cheaper M2M calls
- Operating systems: dual-boot machines, emulators
- Instant messaging technologies: Yahoo! messenger, MSN messenger, Google talk, AIM

Diffusion with Compatibility

1. Each person can adopt multiple behaviors at an added cost.

2. Can adapt to peers with different behaviors.

Benefits of Compatibility



Without compatibility, v can get 2q

… or 3(1 – q)

With compatibility, v can get 2q + 3(1 – q) – c where c is cost of choosing both blue and yellow

Compatibility Model

Let c = rΔ be additional cost of adopting both behaviors (costs r per-edge). Payoff matrix is:

Player 1/ Player 2	A	B	AB
A	(1-q,1-q)	(0,0)	(1-q, 1-q-r)
B	(0,0)	(q,q)	(q, q-r)
AB	(1-q-r, 1-q)	(q-r, q)	(max(q,1-q)-r, max(q,1-q)-r)

Formal Definition

- Infinite Δ-regular graph G
- A strategy profile is a func. s from V(G) to {A,B,AB}
- $s \xrightarrow{v} s'$ if s' is obtained from s by letting v play her best response.
- Similar defn for a finite seq of vertices
- T infinite seq, T_k first k elements of T
- $s \xrightarrow{T} s'$ if for every u, there is $k_o(u)$ such that for every $k>k_o(u)$, $s \xrightarrow{T_k}$ a profile that assigns s'(u) to u.

Definition, cont'd

- For a subset X of V(G), s_X is the profile that assigns A to X and B to V(G)\X.
- A can become epidemic in (G,q,r) if there is
  - a finite set X, and
  - sequence T of V(G)\X

  such that $s_X \xrightarrow{T}$ (all-A).

Basic Facts

Lemma. The only possible changes in the strategy of a vertex are

- from B to A
- from B to AB
- from AB to A.

Corollary. For every set X and sequence T of V(G)\X, there is unique s such that $s_X \xrightarrow{T} s$.

Order Independence

Theorem. If for a set X and some sequence T of V(G)\X, $s_X \xrightarrow{T}$ (all-A), then for *every* sequence T′ that contains every vertex of V(G)\X an infinite # of times, $s_X \xrightarrow{T'}$ (all-A).

Pf idea. T is a subseq of T′. Extra moves make it only more likely to reach all-A.

For which values of (q,r) will new technology become an epidemic?

Partial Answer

Thm [IKMW'07]. A cannot become epidemic in any game (G,q,r) with $q > 1/2$.

Pf idea. Define potential function s.t.

– it is initially finite

– decreases with every best-response move

The following potential function works:

$$q(\#\text{ A-B edges}) + r\Delta(\#\text{ AB vertices})$$

Main Results

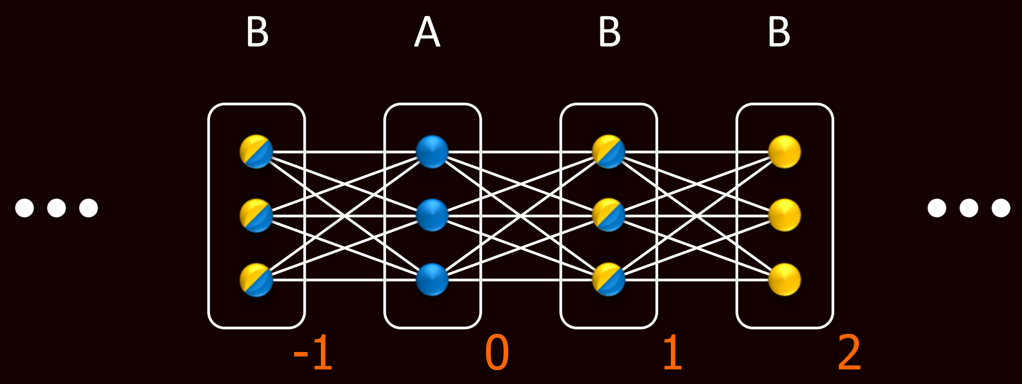
A characterization giving necessary and sufficient conditions for A to spread related to existence of bi-lingual buffers.

A theorem showing that for all graphs G, limited compatibility can help inferior incumbent technologies survive invasion of new superior technology.

Simple Observations

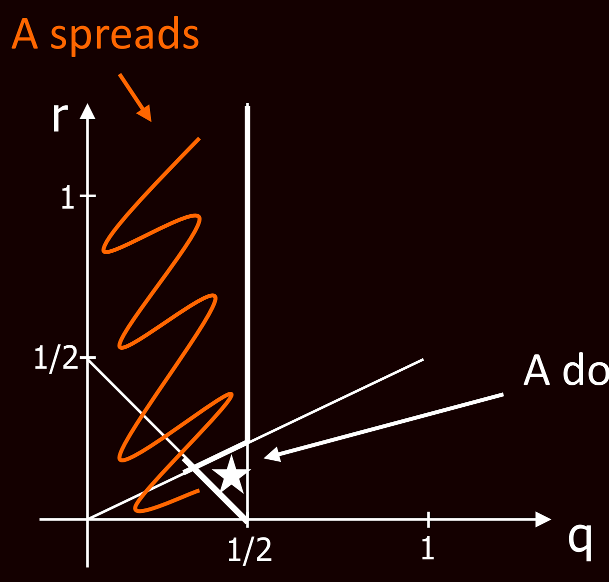
- For high r, technologies are incompatible. Each node will chose just one, and results of Morris carry over.
- For low r, it is almost free to have both technologies. All nodes therefore adopt both and then drop worse one, so contagion happens if $q < ½$.
- For intermediate r?

Example



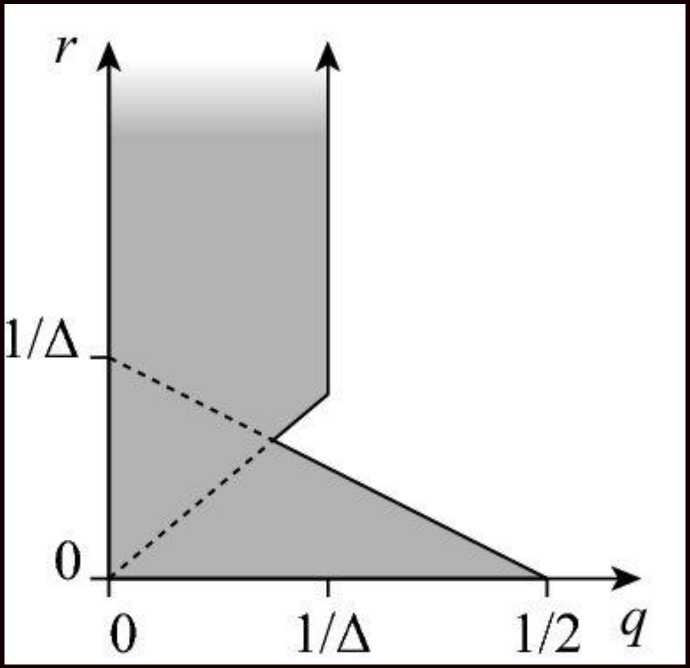
- If r is low, groups 1 and -1 switch to AB to be able to communicate with all neighbors
- … but if r is not low enough, groups 2 and -2 may not find it profitable to adopt A since can already communicate with all 6 neighbors on B!
- For example, $q = 5/12$ and $r = 2/12$

Example

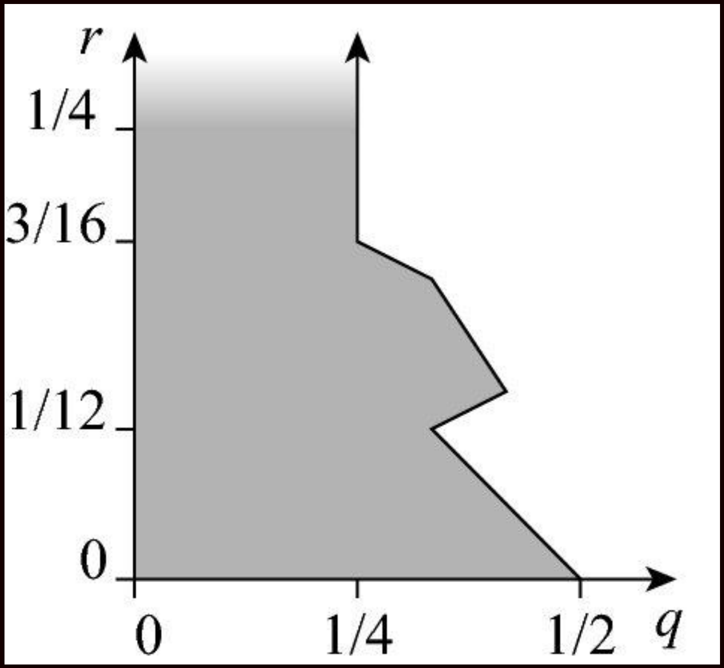


- Technology A can spread if $q < ½$ and either $q+r < ½$ or $2r > q$.

Other Examples



Infinite tree



2-d grid

Interpretation

- Strategically, an inferior incumbant can defend against a new superior option by adopting a limited level of compatibility (e.g., operating system emulators).
- Buffers of bi-lingualism can contain pockets of alternative behaviors, ensuring multiple behaviors will co-exist (e.g., Dutch).

Inferior Incumbants

Can A become epidemic
for every (q,r) with $q < ½$?

Thm [IKMW'07]. For every Δ, there is $q < ½$ and r such that A cannot become epidemic in any (G,q,r).

Assignment:

- Readings:
  - Social and Economic Networks, Chapter 9
  - Bramoulle-Kranton paper; Ballester, Calvo-Armengol and Zenou paper
- Reaction to paper
- Presentation volunteers?