Market Design: Lecture 7

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Recap

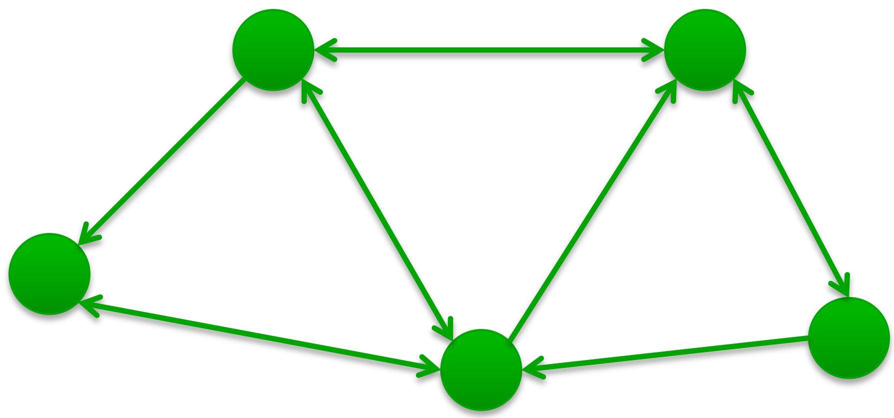
7. discrete allocation: incentives, ordinal efficiency, initial endowments
8. a) kidney exchange: implementation, issues

Outline

8. b) kidney exchange: pairwise exchange, hospital incentives

Part 8: Kidney Exchange.

Pairwise Kidney Exchange



- a set V of patient-donor pairs
- for each pair i, a set N_i of pairs with compatible kidneys (dichotomous prefs)
- a set E of mutually-compatible tuples of pairs

Matching Mechanism

Defn. A matching mechanism for graph G(V,E) outputs subset M of E such that no two edges share a vertex.

... pareto efficient if no other matching makes each patient weakly better off and some patient strictly better off.

... truthful if dominant strategy to reveal full set of acceptable kidneys and potential donors.

Matching Mechanism

Defn. A lottery matching mechanism outputs a distribution over matchings.

... ex-post efficient if distribution over Pareto-efficient matchings.

... ex-ante efficient if no other lottery weakly increases match probability for each agent and strictly for some agent.

Selecting Among Matchings

priority-based: favor matchings that match as many top-priority patients as possible.

egalitarian: pick a lottery that maximizes the probability of match for “poor” patients.

Combinatorial Optimization Aside

- Sets of matchable vertices form a matroid.
- Tutte-Berge formula characterizes size of maximum matching.

Edmonds-Gallai Decompositions

Under-demanded: there is a maximum matching that leaves node unmatched

Over-demanded: not under-demanded yet compatible with an under-demanded node

Perfectly matched: neither under-demanded nor compatible with an under-demanded node.

Priority-Based

Mechanism: Set matched set M to be empty.

For i = 1 to n in order,
… if there's a matching that covers M U {i},
set M = M U {i}.

Theorem. This is Pareto-efficient and truthful.

Egalitarian

- utility is probability of receiving match
- utility profile is Lorenz dominant if
  - for all k, sum of k least fortunate's utilities maximized among all utility profiles

Question. What is upper-bound on utility of least-fortunate patient?

Egalitarian

D_i = odd comps, $D = U_i D_i$, N^O = over-dem. nodes

Repeat until done:

- D is remaining odd comps, N^O remaining nodes
- $J_k = \text{argmin}_{J \text{ in } D} \text{util}(J, N^O)$
- $N_k^O = \text{Neighbors}(J_k, N^O)$

See example on board.

Efficiency of Short Cycle Exchanges

Recall blood-type compatibility:

- O can donate to any type
- A can donate to A or AB
- B can donate to B or AB
- AB can only donate to AB

O patients disadvantaged, O kidneys short supply.

3-Way vs 2-Way Exchange

- better use of O-donors
- helps highly sensitized patients
- allows same-type patient-donor pairs (e.g., A-A) to trade with pairs of other types

Limitation of 3-Way Exchanges

Improvement over 2-way:

- Enable odd # of same-type transplants
- O donors can facilitate 3 transplants

However, AB-O types still problematic (but rare, i.e., only 3.85 percent of database).

4-Way Exchanges Suffice

Assumption 1 (large markets): No patient tissue-type incompatible with another patient's donor.

Assumption 2: >1 of each same-type pair.

Assumption 3: Any maximum matching leaves at least one of each "long-type" unmatched.

Theorem. Every efficient matching can be carried out with 4-cycles.

Greg's Presentation:
Incentives for Hospitals