Market Design: Lecture 6

NICOLE IMMORLICA, NORTHWESTERN UNIVERSITY

Recap

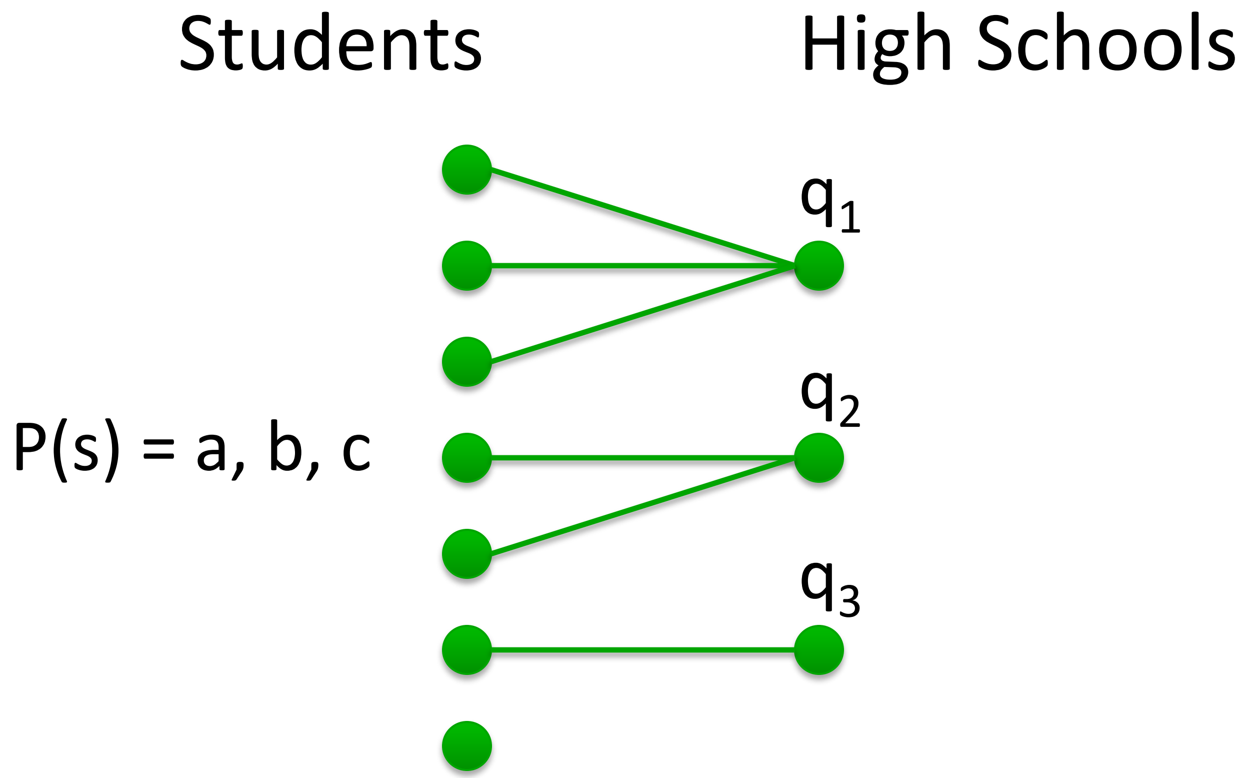
6. b) Large market results: incentives, couples

Outline

7. discrete allocation: incentives, ordinal efficiency, initial endowments
8. kidney exchange: implementation, issues

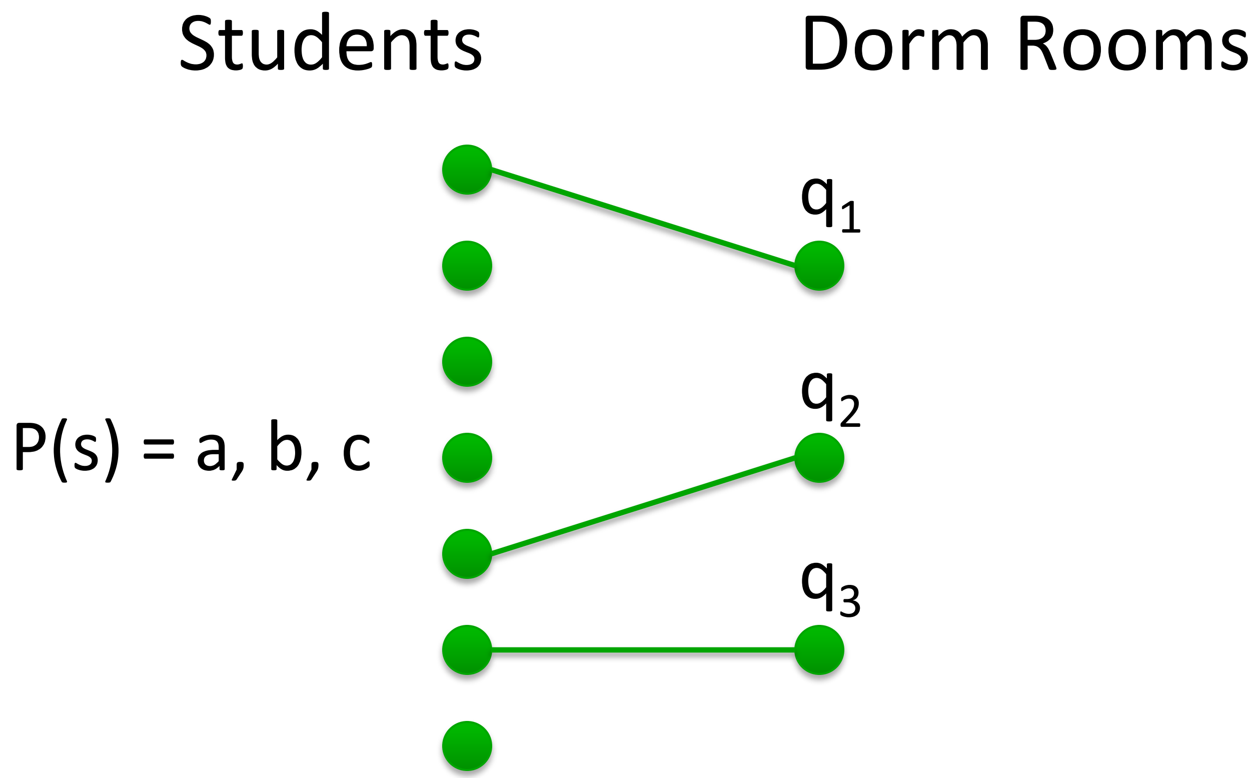
Part 7: Discrete Allocation.

Motivation: School Choice



each student has a preference list P(.) over schools

Motivation: Housing



each student has a preference list P(.) over rooms

Model

Agents Items

1

P(i)

m

matching μ strongly Pareto efficient if there is no other matching ν such that

- $\nu(i) \geq_i \mu(i)$ for all agents i,
- and $\nu(i) >_i \mu(i)$ for at least one agent i

- market consists of agents, items, and preference list P(i) over items for each agent i
- assignment is a matching μ assigning each agent at most one item and each item to at most one agent

Serial Dictatorship

- Fix priority function π which specifies a permutation of agents.
- For i = 1 to n, set match of $\pi(i)$ equal to his favorite remaining item.

Serial Dictatorship

- agents {1, 2, 3, 4}, items {a, b}
  - agents 1 and 2 prefer a to b
  - agents 3 and 4 prefer b to a
- priority $\pi(i) = i$:
  1. agent 1 selects item a, only b remains
  2. agent 2 selects item b, no items remain
  3. agents 3 and 4 are unmatched

Serial Dictatorship

Theorem. Serial dictatorship is truthful and Pareto efficient.

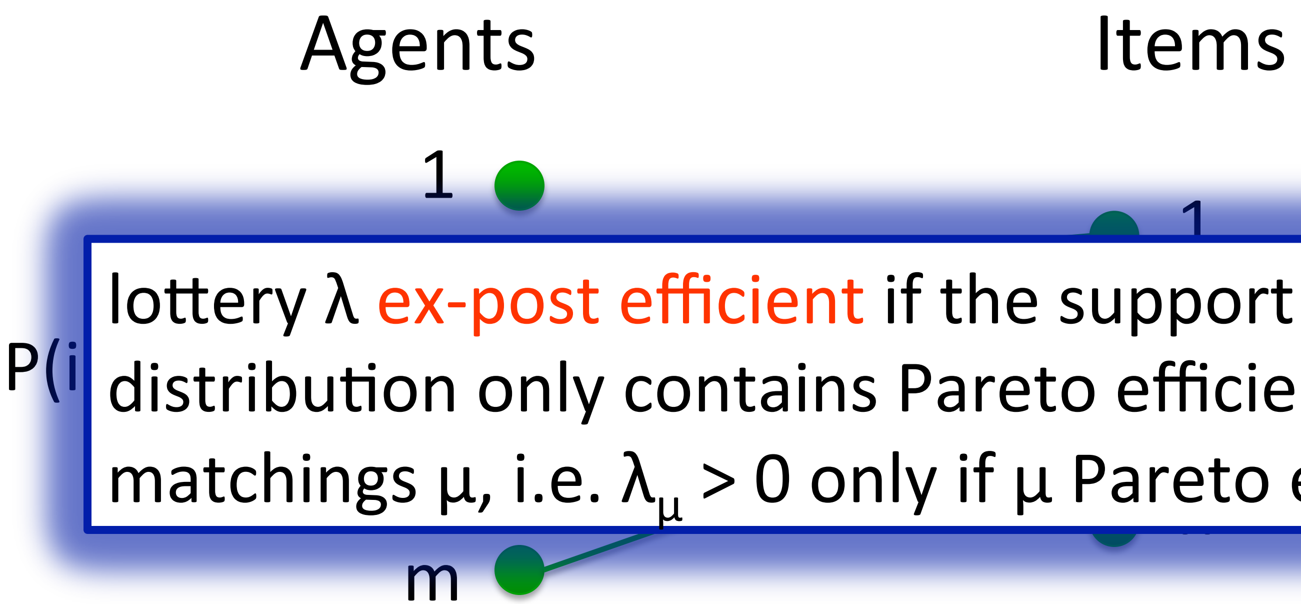
In fact, it is the *only* mechanism that is truthful and satisfies other natural properties

- neutral: renaming items doesn't affect match
- non-bossy: a manipulating agent can only change the match if his own match changes

Objection:

Serial dictatorship is not *fair*!

Lottery Mechanisms



lottery λ ex-post efficient if the support of the distribution only contains Pareto efficient matchings μ, i.e. $\lambda_\mu > 0$ only if μ Pareto efficient.

- assignment is a matching μ assigning each agent at most one item and each item to at most one agent
- lottery is a probability distribution $\lambda = (\lambda_\mu)$ over matchings μ such that $\Sigma_\mu \lambda_\mu = 1$

Random Assignment

$$X = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 2/3 & 1/6 & 1/6 \\ 0 & 1/2 & 1/2 \end{pmatrix}$$

x_{ij} = Pr[agent i is matched to item j]

X is *bi-stochastic*.

Lotteries as Random Assignments

Lottery λ

$$x_{ij} = \Sigma_{\mu\,:\,\mu(i)=j}\lambda_\mu$$

Random assignment X

Random Assignments as Lotteries

Lottery λ

Birkoff-von Neumann Theorem

Random assignment X

Birkoff-von Neumann

Theorem. Any bi-stochastic matrix can be written as the convex combination of permutation matrices (i.e., matchings).

$$\begin{pmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1/2 & 1/2 \end{pmatrix} = (1/2) \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + (1/2) \times \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Fair Lottery

Random Dictatorship: Run serial dictatorship for random priority ordering.

Example: agents {1, 2, 3, 4}, items {a, b}
preferences P(1) = P(2) = a, b; P(3) = P(4) = b, a

Random Dictatorship	Item a	Item b	unmatched
Agents 1, 2	5/12	1/12	1/2
Agents 3, 4	1/12	5/12	1/2

Properties

- truthful? ✓
- ex-post efficient? ✓
- fair?

equal treatment of equals: agents with same preference receive same distribution of items.

Inefficiency

Random Dictatorship	Item a	Item b	unmatched
Agents 1, 2	5/12	1/12	1/2
Agents 3, 4	1/12	5/12	1/2

recall: P(1) = P(2) = a, b; P(3) = P(4) = b, a

Preferred Assignment	Item a	Item b	unmatched
Agents 1, 2	1/2	0	1/2
Agents 3, 4	0	1/2	1/2

Ordinal Efficiency

Defn. Random assignment X ordinally dominates random assignment Y if for every agent i, X_i stochastically dominates Y_i:

Pr[i gets item weakly preferred to a in X]

≤ Pr[i gets item weakly preferred to a in Y]

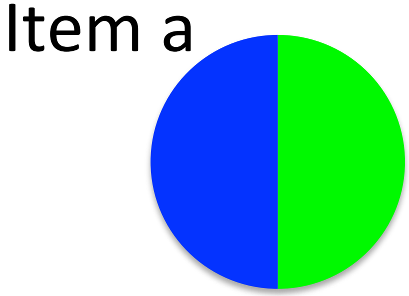
for all i, a.

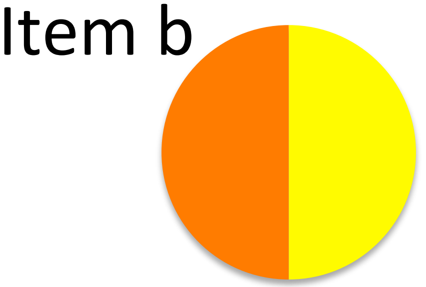
Eating Mechanism

(aka probabilistic serial)

- each agent eats favorite pie at constant rate
- eaten fractions form random assignment

Example: agents {1, 2, 3, 4}, items {a, b}
preferences P(1) = P(2) = a, b; P(3) = P(4) = b, a





Inefficiency

Random Dictatorship	Item a	Item b	unmatched
Agents 1, 2	5/12	1/12	1/2
Agents 3, 4	1/12	5/12	1/2

recall: P(1) = P(2) = a, b; P(3) = P(4) = b, a

Eating Mechanism	Item a	Item b	unmatched
Agents 1, 2	1/2	0	1/2
Agents 3, 4	0	1/2	1/2

Properties

- ordinally efficient? ✓
- fair?

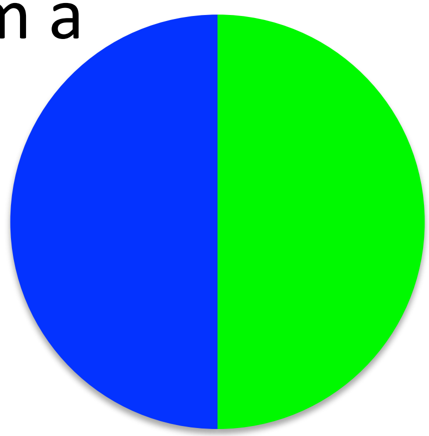
Envy-free: everyone likes his/her assignment better than anyone else's.

- truthful? ✗

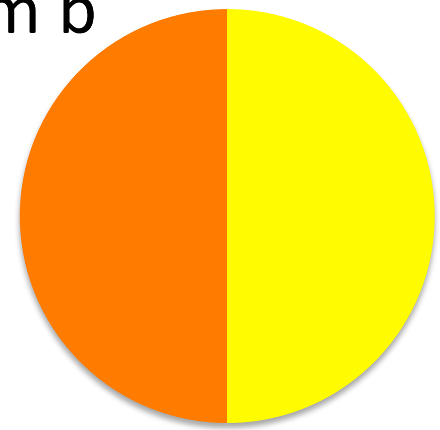
Eating Mechanism Not Truthful

Example: agents {1, 2, 3, 4}, items {a, b}
preferences P(1) = a, b; P(2) = a; P(3) = P(4) = b

Item a



Item b



Eating Mechanism Not Truthful

Example: agents {1, 2, 3, 4}, items {a, b}

preferences P’(1) = b, a; P(2) = a; P(3) = P(4) = b

time t = 1/3:

Item a　　Item b

time t = 1:

Item a　　Item a

Eating Mechanism Not Truthful

True Preferences	Item a	Item b	unmatched
Agents 1	1/2	0	1/2

deviation beneficial for some cardinal preferences consistent with ordinal ones

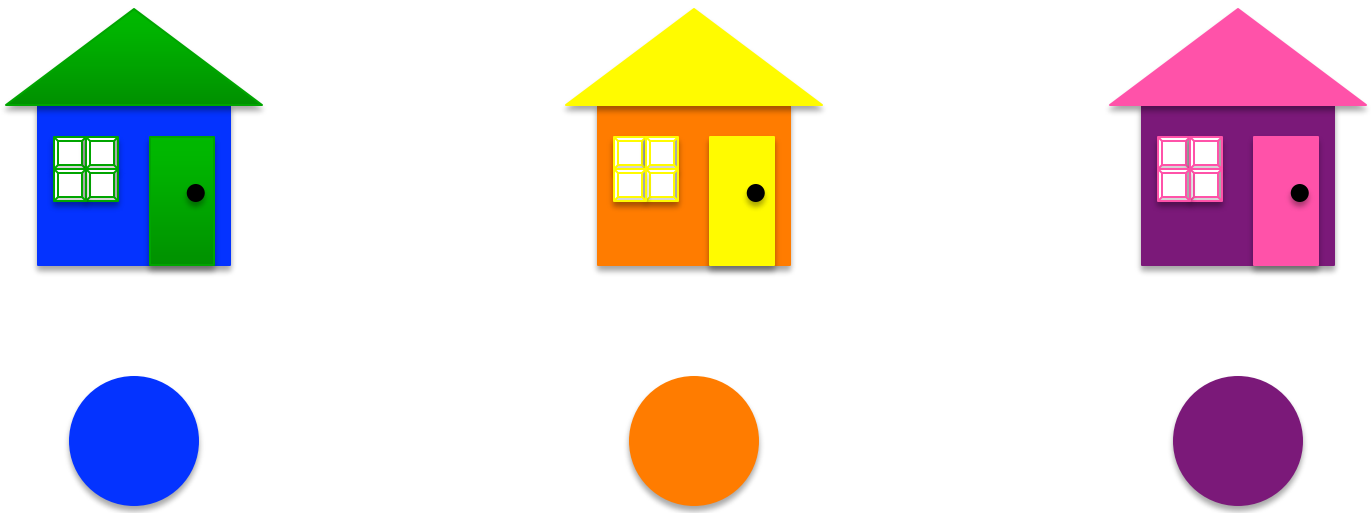
Altered Preferences	Item a	Item b	unmatched
Agents 1	1/3	1/3	1/3

Ordinal Efficiency and Truthfulness

Theorem: There is no mechanism that is ordinally efficient, truthful, and satisfies equal treatment of equals.

Theorem [Che-Kojima ’09]: As the market “grows,” the random assignment from random dictatorship and eating mechanism converge.

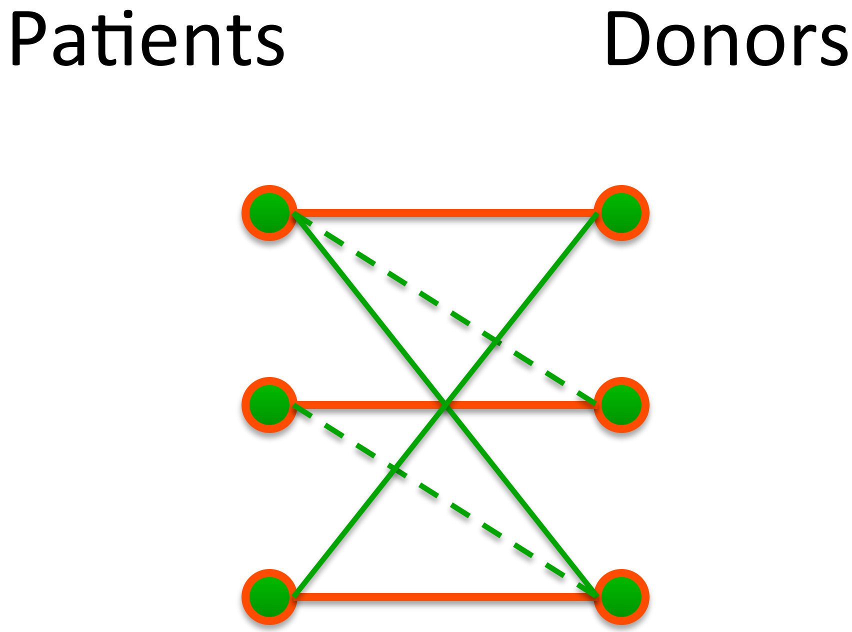
Endowments



Housing market:

- agents $A = \{a_1, \ldots, a_n\}$, houses $H = \{h_1, \ldots, h_n\}$ (agent a_i owns house h_i)
- set of preferences $P(a_i)$ of agents over houses

Motivation: Kidney Exchange



each patient has a (strict) preference list P(.) over kidneys

Stability Concept

Defn. A matching μ is in the core if no group of agents can profitably deviate, i.e., there is no matching ν and coalition B s.t. for every a in B

- ν(a) initially owned by some a' in B
- $\nu(a) \geq_a \mu(a)$ and for some a in B, $\nu(a) >_a \mu(a)$

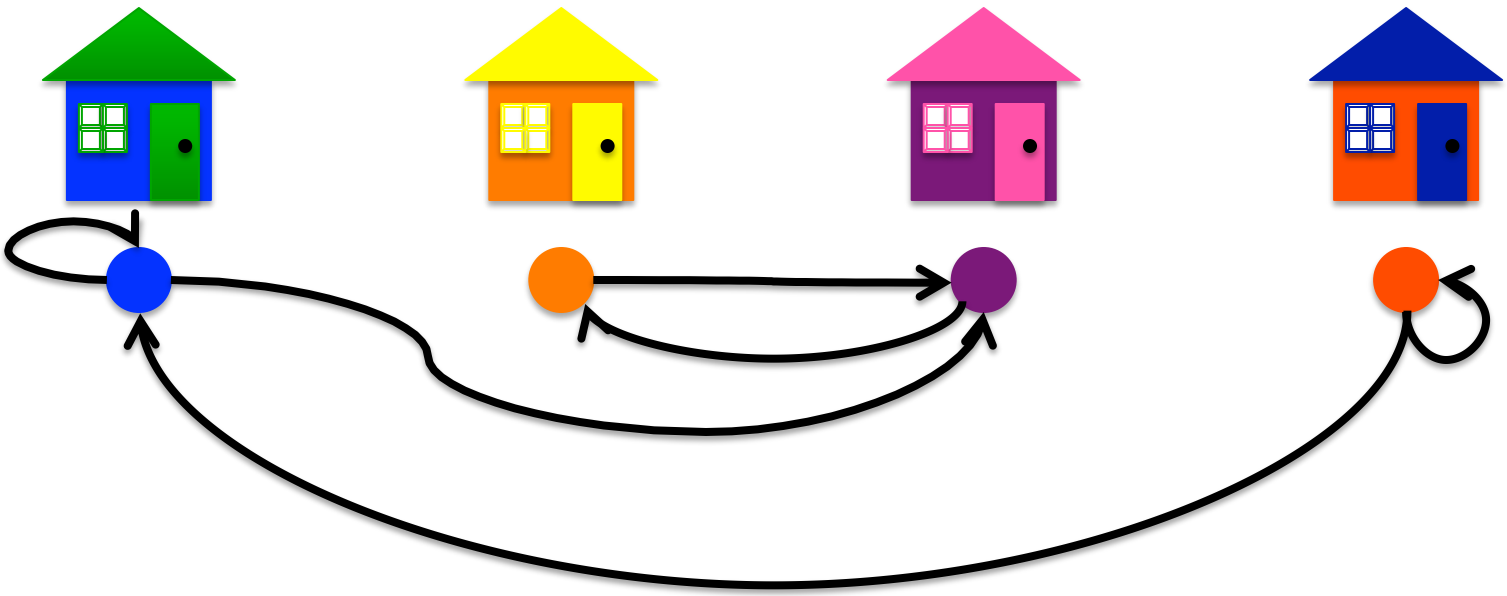
Compare to Pareto-efficiency.

Properties of Core

- Individually rational (every agent receives house at least as good as initial house): consider coalition of single agent
- Pareto optimal: consider coalition of all agents

Theorem. Core exists for every housing market.

Top Trading Cycles



1. ask each agent to point to favorite house
2. choose cycle, perform trades, remove match
3. repeat

Properties

- truthful? ✓
- in the core? ✓

Theorem. TTC produces unique point in core!

Theorem. Mechanism is truthful, IR, Pareto-efficient iff it is TTC.

House Allocation with Tenants

Generalize house allocation and house market

- Agents: existing tenants or newcomers
- Houses: currently owned or vacant

Used for dorm assignment at CMU, Duke, etc.:

- Existing tenants choose to participate or not
- Serial dictatorship run on all participants

Desirable Properties

- Pareto efficiency
- Strategy-proofness
- Individual rationality

All jointly achieved by

- Serial dictatorship (in house allocation)
- Top trading cycles (in house market)

Proposed Mechanism

You-Get-My-House, I-Get-Your-Turn:

- Fix ordering (can be chosen randomly)
- Let agents select favorite houses in order until someone asks for a house with existing tenant
- If existing tenant already got a house, proceed
- Else insert tenant at top of priority order
- Clear cycles as they form

Example

- Tenants $a_1, \ldots, a_9$ occupying houses $h_1, \ldots, h_9$
- Newcomers $a_{10}, \ldots, a_{16}$, vacancies $h_{10}, \ldots, h_{16}$
- Preferences:

a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
h_{15}	h_3	h_1	h_2	h_9	h_6	h_6	h_6	h_{11}
	h_4	h_3				h_7	h_{12}	

a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}
h_7	h_2	h_4	h_6	h_8	h_1	h_5
h_3	h_4	h_{14}	h_{13}			
h_{12}	h_{16}					
h_{10}						

- Priority ordering:

$a_{13}, a_{15}, a_{11}, a_{14}, a_{12}, a_{16}, a_{10}, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9$

Properties

Theorem. Mechanism is individually rational, strategy-proof, and Pareto efficient.

Theorem. It is only IR, strategy-proof, PE, neutral and consistent mechanism.

(consistent: alloc. same when run on submarket)

Part 8: Kidney Exchange.

Kidney Transplant

Transplant as treatment of kidney disease:

- Over 70000 patients are on waiting lists for kidney in the U.S.
- In 2006, there were
  - 10659 transplants from diseased donors,
  - 6428 transplants from living donors, while
  - 3875 patients died while on the waiting list.

Kidney Sales

See YouTube video.

Kidney Donation

- Buying /selling kidneys is illegal in the U.S.

  "it shall be unlawful for any person to knowingly acquire, receive or otherwise transfer any human organ for valuable consideration for use in human transplantation."

  Section 301 of the National Organ Transplant Act

- Donation is most important source of kidneys.

Donation Sources

There are two sources of donation:

- Deceased donors: A centralized mechanism has been used for allocation of deceased donor kidneys.
- Living donors: Living donors usually come from friends or relatives of a patient (because the monetary transaction is prohibited). Live donation has been increasing recently.

Donor Types	2008	1998	1988
All donors	10,920	9,761	5,693
Deceased	5,992	5,339	3,876
Live	4,928	4,422	1,817

Donor Compatibility

Donor/patient must be compatible:

- Blood type: O, A, B, AB.
  - O type patients can receive kidneys from O type
  - A type patients can receive kidneys from O or A type
  - B type patients can receive kidneys from O or B type
  - AB type patients can receive kidneys from any type
- HLA tissue compatibility (blood proteins)

Increasing Successful Transplants

- Paired exchange: incompatible patient/donor pairs can swap donors
- List exchange: match one incompatible patient/donor pair and deceased donor list

Kidney Exchange Clearinghouse

- Renal Transplant Oversight Committee of New England approved establishment of clearinghouse for kidney exchange (2004)
- Economists (Roth, Sonmez, Unver) as well as doctors designed the clearinghouse.
- Potential issues include
  - Efficiency (Pareto efficiency; maximizing number of transplantation)
  - Incentives (Strategy-proofness)
  - Fairness

Incentives in Kidney Market

A news report by Reuters (2003-7-29)

Three Chicago hospitals were accused of fraud by prosecutors on Monday for manipulating diagnoses of transplant patients to get them new livers. Two of the institutions paid fines to settle the charges. "By falsely diagnosing patients and placing them in intensive care to make them appear more sick than they were, these three highly regarded medical centers made patients eligible for liver transplants ahead of others who were waiting for organs in the transplant region," said Patrick Fitzgerald, the U.S. attorney for the Northern District of Illinois.

Kidney Exchange Model

- A set of donor-patient pairs $\{(k_1, t_1), \ldots, (k_n, t_n)\}$
- A preference over $\{k_1, \ldots, k_n\}$ U $\{w\}$ for each t_i (where w is priority in waitlist)
- Matching: match patients to kidneys; any number of patients to waitlist
- Mechanism: procedure to select matching

Design 1

Roth, Somnez, Unver (2004)

- No limit on # pairs in one exchange
- Preferences are strict

Solution: use house allocation with tenants

- (tenant, house) = (patient, donor)
- Good Samaritan donors = vacant houses
- Patients w/out donor = newcomers

Design 1

Dealing with waitlist w:

- Top trading cycles, point to kidney or waitlist
- At any step, there’s either a cycle or a chain
- Remove cycle if exists

How to choose chains?

- Min/max/priority-based
- Leave agents present/remove them

Example

t_1: k_9, k_{10}, k_1	t_7: k_6, k_1, k_3, k_9, k_{10}, w
t_2: k_{11}, k_3, k_5, k_6, k_2	t_8: k_6, k_4, k_{11}, k_2, k_3, k_8
t_3: k_2, k_4, k_5, k_6, k_7, k_8, w	t_9: k_3, k_{11}, w
t_4: k_5, k_9, k_1, k_8, k_{10}, k_3, w	t_{10}: k_{11}, k_1, k_4, k_5, k_6, k_7, w
t_5: k_3, k_7, k_{11}, k_4, k_5	t_{11}: k_3, k_6, k_5, k_{11}
t_6: k_3, k_5, k_8, k_6	t_{12}: k_{11}, k_3, k_5, k_9, k_8, k_{10}, k_{12}

Run with longest-chain selection rule.

Incentives: Suppose t_4 promotes k_1 to 2^{nd} place.

Design 1

- Pareto-efficient if “keep” chains until end.
- Strategy-proof if choose minimal or based on priority of head of list.