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Public goods in networks

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Abstract

This paper considers incentives to provide goods that are non-excludable along social or geographic links. We find, first, that networks can lead to specialization in public good provision. In every social network there is an equilibrium where some individuals contribute and others free ride. In many networks, this extreme is the only outcome. Second, specialization can benefit society as a whole. This outcome arises when contributors are linked, collectively, to many agents. Finally, a new link increases access to public goods, but reduces individual incentives to contribute. Hence, overall welfare can be higher when there are holes in a network.

JEL classification: D00; D83; H41; O31

Keywords: Strategic substitutes; Independent sets; Experimentation; Information sharing

1. Introduction

This paper builds a network model of public goods. We examine the incentive to provide goods that are non-excludable along social or geographic links. Examples include several classic public goods. When a person plants a garden, her neighbors benefit. When a jurisdiction institutes a pollution abatement program, the benefits also accrue to nearby communities. Our primary working example in this paper is innovation. Individuals innovate—e.g., experiment with new technology or generate new information—and the results are often non-excludable in certain dimensions. We see this public goods nature of innovation and information in many areas of economics. Consumers benefit from research of friends and family into new products (e.g., [23]). In medicine and other technical fields, professional networks shape research patterns (e.g., [18,14,37]). In

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agriculture, one farmer's experience with a new crop can benefit other farmers, and the physical and social geography of the countryside can influence experimentation and learning (e.g., [24,19,35]). In industry, it has been long posited that research findings spillover to other firms. Information spread is often local and thus can depend on social networks and the geography of industry and trade (e.g., [33,13,31]). In all these settings, social structures and geography can influence the incentive to innovate.

The local nature of public goods and spread of information raises a new set of research questions. How does the social or geographic structure affect the level and pattern of public good provision? Do people exert effort themselves or rely on others? How do new links—links between communities or firms, for example—affect contributions and welfare?

This paper builds a model to address these questions. There is fixed social/geographic structure. People desire a good which is costly to produce. This good is non-excludable among linked individuals. Individuals decide how much to contribute to this good, knowing the good is non-excludable in this way.

Our analysis yields three main insights.

First, networks can lead to specialization. In any network there is an equilibrium where some individuals contribute to the public good and others completely free ride. In many networks, this extreme is the only equilibrium outcome. In all networks, such patterns are the only stable outcomes. Hence, agents' positions in a network can determine whether or not they contribute to the public good.

Second, specialization can have welfare benefits. This outcome arises when contributors are linked, collectively, to many people in society.

Finally, new links can reduce overall welfare. A new link increases access to the new information/public good, but also reduces individual incentives to contribute. Hence, overall welfare can be higher when there are holes in a network.

This paper contributes to several research areas.

First, it introduces the first network model of public goods. We motivate our model with innovation and information. The model applies to any good that is non-excludable in a geographic or social dimension. Related work includes [21] who study public goods in interconnected ethnic communities. Bloch and Zenginobuz [9] examine local public goods with geographic spillovers across jurisdictions. We study the provision of a public good embedded in a general network structure. [1]

Second, the paper advances a new model of innovation and social learning. The two key elements of our paper—the generation of new information and social networks—do not appear together in existing social learning theory. [2] In our analysis, individuals must pay a cost to gain new information—that is, private signals are not free. In addition, this new information is a public good among linked individuals. The model thus considers strategic experimentation, as in [24,10]

[1] There is a large literature on local public goods, i.e., public goods that are accessible only to residents of a jurisdiction. Models typically involve individuals choosing where to locate knowing that everyone in the same jurisdiction benefits from the same level of public good. In our model, there is no location choice. Rather, individuals can have access to different levels of the public good depending on their position in the network and on the contributions of their direct neighbors.

[2] For review of the social learning literature see [8,17]. Only a few papers consider social networks: Bala and Goyal [2,3] and Cowan and Jonard [20] model agents learning the choices and payoffs only of linked individuals.

who consider individual incentives to innovate when results are shared by others. [3] Our innovation is the social network.

Finally, this paper contributes to the economic theory of networks. [4] We consider a game where agents take actions that are substitutes with their neighbors' actions. Our analysis will apply to any such setting. [5] We relate the Nash equilibria and the stable equilibria of this game to a graph-theoretic notion—maximal independent sets. An *independent set* of a graph is a set of agents such that no two agents who belong to the set are linked. We show that maximal independent sets are a natural notion in network strategic-substitutes games. Because of strategic substitutability, agents who specialize cannot be linked to each other in equilibrium. Hence, they constitute an independent set. We show that equilibria where some agents contribute and other agents free ride always exist and correspond to this structural property of a graph. Moreover, only these equilibria are stable.

The paper is organized as follows. In the next section, we present the model and in Section 3, we study the Nash and stable equilibria of the game. In Section 4, we study economic welfare for a given graph, and in Section 5 we ask how changing the graph structure can affect welfare. In Section 6, we discuss the robustness of our findings to changes in the model's specifications. Section 7 concludes.

2. The model

2.1. Public goods in a network

There are n agents, and the set of agents is $N = \{1, \ldots, n\}$. Let $e_i \in [0, +\infty)$ denote agent i's level of effort. E.g., e_i could be the amount of land dedicated to a new crop, as in [24], or e_i could be the amount of time a consumer spends researching a new product. We assume the individual marginal cost of effort is constant and equal to c. Let $\mathbf{e} = (e_1, \ldots, e_n)$ denote an effort profile of all agents.

Agents are arranged in a network, which we represent as a graph $\mathbf{g}$, where $g_{ij} = 1$ if agent j benefits directly from the results of agent i's effort, and $g_{ij} = 0$ otherwise. We assume that results flow both ways so that $g_{ij} = g_{ji}$. Since agent i knows the results of his own effort, we set $g_{ii} = 1$. Let N_i denote the set of agents that benefit directly from agent i's efforts, called i's *neighbors*: $N_i = \{j \in N \backslash i : g_{ij} = 1\}$. Let $k_i = |N_i|$ denote the number of agent i's neighbors. Agent i's *neighborhood* is defined as himself and his set of neighbors; i.e., $i \cup N_i$.

We make two important assumptions concerning the substitutability of agents' efforts. First, an agent's effort is a substitute of the efforts of her neighbors, but not of individuals further away in the graph. We make this assumption for simplicity and because it reflects findings that information

[3] Foster and Rosenzweig [24] document the public goods nature of experimentation in their study of high-yield varieties in India. Bolton and Harris [10] study dynamics of experimentation—how early experimentation leads to more or less future experimentation.

[4] Much work on networks considers the properties of equilibrium networks. For a review see [22]. Research in specific economic settings includes coordination [38,12], job market networks [11,16] and firms [34,29].

[5] Recently [4,5,27] have developed results that apply to our setting. Ballester and Calvó-Armengol [4], Ballester et al. [5] focus on situations where the equilibrium is unique. Galeotti et al. [27] study different informational assumptions.

does not travel more than one or two steps in a network [25]. [6] Second, a neighbor's effort is a perfect substitute with one's own. (We relax this assumption in Section 6.) With these assumptions an agent i derives benefits from the total of his own and his neighbors efforts.

We assume each agent receives benefits from own and neighbors' effort according to a (twice-differentiable) strictly concave benefit function $b(e)$ where $b(0) = 0$, $b' > 0$ and $b'' < 0$. With our assumptions above, an individual i has benefits $b(e_i + \sum_{j\in N_i} e_j)$.

An agent i's payoff from profile $\mathbf{e}$ in graph $\mathbf{g}$ is then

$$U_i(\mathbf{e};\mathbf{g}) = b\left(e_i + \sum_{j\in N_i} e_j\right) - ce_i.$$

To fix ideas for these payoffs, consider the following discrete example. Suppose agents want to know the best production technique. There are many techniques, each with a different unknown value of output. The values are distributed according to a distribution F. Let e_i be the number of i's draws, at cost c per draw. [7] Agent i's final information set consists of $e_i + \sum_{j\in N_i} e_j$ trials of different technologies. When all trials are independent, the expected benefit is the expectation of the first-order statistic of e trials. The benefits $b(e)$ are then increasing and concave in e.

In Section 6, we discuss the robustness of our findings to alternative specifications of the payoffs, including convex costs. A main interest of our simple quasi-linear form (besides its analytic tractability) is that it allows us to focus on network structure.

2.2. *Strategic interaction*

We specify the following game. Given a structure $\mathbf{g}$, agents simultaneously choose effort levels. For a profile $\mathbf{e}$, each agent i earns payoffs $U_i(\mathbf{e};\mathbf{g})$. We analyze pure strategy Nash equilibria, as there are no mixed strategy equilibria. [8] In the following analysis, we explore how network structure influences the equilibrium effort levels.

3. Equilibrium contributions to public goods in a network

3.1. *The shape of equilibrium profiles*

We first characterize the Nash equilibria. Let e^* denote the effort level at which, to an individual agent, the marginal benefit equals its marginal cost; $b'(e^*) = c$. [9] Let $\overline{e}_i = \sum_{j\in N_i} e_j$ be the total effort of i's neighbors.

A profile $\mathbf{e}$ is a Nash equilibrium if and only if for every agent i either (1) $\overline{e}_i \geqslant e^*$ and $e_i = 0$ or (2) $\overline{e}_i \leqslant e$ and $e_i = e^* - \overline{e}_i$. The argument is straightforward. Agents want to exert effort as long

[6] The methods we develop here can be extended to diffusion of more than one step. Suppose that efforts benefit agents k steps away in the graph. Define the graph $\mathbf{g}^{(k)}$ as follows: $g_{ij}^{(k)} = 1$ if i and j are less than k-step apart in $\mathbf{g}$, and 0 otherwise. Under the assumption that agents can discern redundant contributions, k-step diffusion on the graph $\mathbf{g}$ is formally equivalent to 1-step diffusion on $\mathbf{g}^{(k)}$ and our analysis directly extends. Decay along links would change the analysis, as effort exerted in one part of the graph eventually reaches all agents in the graph.

[7] For simplicity, the agent is making these draws with replacement.

[8] Since the benefit function $b(.)$ is concave and costs are linear, an agent would always earn higher expected payoffs by playing the average of a set of effort levels than a mixture over the set of effort levels. Hence, there is no Nash equilibrium where an agent plays a mixture of effort levels.

[9] Given $b(.)$ is strictly concave, a level of search $e^* > 0$ exists and is well-defined as long as $b'(0) > c$.

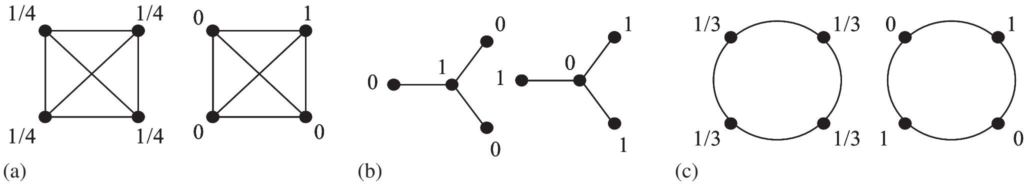

Fig. 1. Equilibria in basic graphs with four agents.

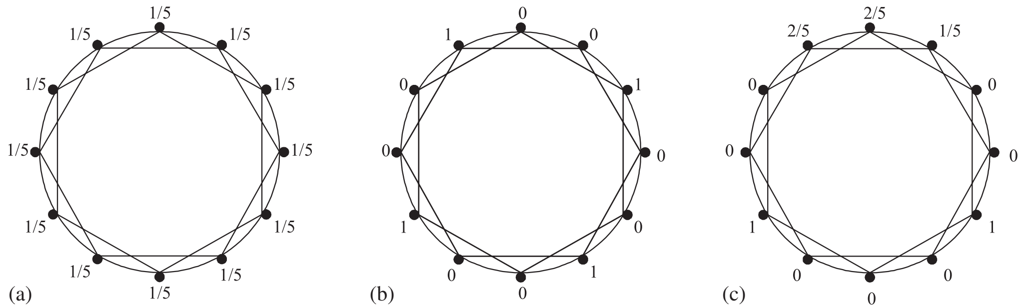

Fig. 2. Equilibria in local interaction graph.

as their total benefits are less than $b(e^*)$. Thus, if the benefits they acquire from their neighbors are more than $b(e^*)$, they exert no effort. If the benefits are less than $b(e^*)$, they exert effort up to the point where their benefits equal $b(e^*)$.

In this game, effort levels are strategic substitutes. The more effort an agents' neighbors exert, the less an agent exerts himself. We say a profile **e** is *specialized* when every agent either exerts the maximum amount of effort e^* or exerts no effort; for all agents i either $e_i = 0$ or $e_i = e^*$. We call an agent who exerts e^* a *specialist*. We say a profile **e** is *distributed* when every agent exerts some effort; for all agents i, $0 < e_i < e^*$. *Hybrid* equilibria fall between these two extremes.

The following example illustrates different kinds of Nash equilibria. Note that in many figures we set $e^* = 1$ for ease of exposition.

Example 1. Nash equilibria. Fig. 1 shows the complete graph, the circle, and the star for four agents. In the complete graph, in any equilibrium, aggregate effort is e^*, and it can be split in any way among the agents. E.g., effort could be equally distributed, so that each agent exerts $\frac{1}{4}e^*$, or one agent could be a specialist, as in panel (a). On the star, only specialized profiles are equilibria. There are just two Nash equilibria: either the center is a specialist, or the three agents at the periphery are specialists, as shown in panel (b). Finally, on the circle, effort can be distributed among the agents, or concentrated among specialists, as shown in panel (c). Fig. 2 shows a local interactions graph. For any agent, in any equilibrium, there must be at least an aggregate effort of e^* in the agent's neighborhood. Effort could be equally distributed, so that each agent make some contribution, as in panel (a). Agents can also be specialists, and other agents completely free ride, as in panel (b). Finally, in hybrid equilibria, some agents specialize, some agents make smaller contributions, and other agents free ride, as shown in panel (c).

3.2. The existence of Nash equilibria: specialized profiles and independent sets

In this section we show that for every graph there exists a specialized equilibrium. [10] Our proof is constructive and provides a method to find all specialized equilibria in general graphs.

We use a concept from graph theory—maximal independent sets. An *independent set I* of a graph **g** is a set of agents such that no two agents who belong to I are linked; i.e., $\forall i, j \in I$ such that $i \neq j$, $g_{ij} = 0$. An independent set is *maximal* when it is not a proper subset of any other independent set.

We use the following properties of maximal independent sets. Given a maximal independent set I, every agent either belongs to I or is connected to an agent who belongs to I. [11] Thus, we can partition the population into two disjoint sets of agents: those who belong to maximal independent set I, and those who are linked to an agent in I. For any agent i, there exists a maximal independent set I of the graph **g** such that i belongs to I. [12] Mathematicians and computer scientists have derived several results concerning maximal independent sets which we do not use here but could be useful in applications (see e.g. [32]). [13]

Given a graph **g**, define a *maximal independent set of order r* as a maximal independent set I such that any individual not in I is connected to at least r individuals in I. That is, for a maximal independent set of order r, agents outside the set can have more than r, but no less than r, connections to agents in the set. The case $r = 1$ simply corresponds to maximal independent sets. While every graph contains maximal independent sets, not all graphs contain maximal independent sets of higher order. (For example, in the complete graph, there is no maximal independent set of order $r = 2$).

Maximal independent sets are a natural notion in our context. Because efforts are strategic substitutes, in equilibrium no two specialists can be linked. Hence, specialized equilibria are characterized by this structural property of a graph:

Theorem 1. *A specialized profile is a Nash equilibrium if and only if its set of specialists is a maximal independent set of the structure* **g**. *Since for every* **g** *there exists a maximal independent set*, *there always exists a specialized Nash equilibrium.*

Proof. All proofs are provided in the Appendix.

The next example illustrates the concept of maximal independent sets and the relationship to specialized equilibria. We use the basic graphs in Figs. 1 and 2.

[10] There exists a Nash equilibrium profile for any social structure. It is easy to show that the best-response function is continuous from the compact convex set $\{\mathbf{e} \in R^n : \forall i, 0 \leqslant e_i \leqslant e^*\}$ to itself. The result follows from Brouwer's Fixed Point Theorem.

[11] To see this, suppose not. Let I be a maximal independent set, and let i be an agent who does not belong to I and is not connected to any agent who belongs to I. Then the set $I \cup \{i\}$ is an independent set, and hence I is not maximal.

[12] To see this, note that i itself is an independent set. To build a maximal independent set, begin with i and successively add agents not linked to i, then agents not linked to those agents, etc.

[13] In particular, with n nodes, the number of maximal independent sets is (weakly) lower than $3^{n/3}$. There exists an algorithm that lists all maximal independent sets of a graph running in time $O(3^{n/3})$, where an algorithm runs in time $O(h(n))$ if there exists a constant K such that the algorithm gives the answer with at most $Kh(n)$ operations. Hence, listing all maximal independent sets can be done in exponential time, but not in polynomial time. The problems of finding the largest and the smallest maximal independent sets are both NP-hard. The first of these two problems has been well-studied. For instance, there exists an algorithm that finds the largest independent set of a graph in time $O(2^{0,276n})$. There exist algorithms and results for specific families of graphs (trees, bipartite, etc.). The size of the largest maximal independent set is always greater than or equal to $\sum_{i \in N} \frac{1}{k_i+1}$.

Example 2. Specialized equilibria and maximal independent sets. In a complete graph, an independent set can include at most one agent. Hence, for $n = 4$ there are four specialized equilibria, corresponding to each agent. On the star, there are two maximal independent sets: the agent at the center, and the three agents in the periphery. These two sets correspond to the two specialized equilibria (and only equilibria) for the star. In the circle, there are two maximal independent sets, each containing two agents on opposite sides of the circle. Again, these two sets correspond to the specialized equilibria for the circle. We can again see a maximal independent set in the overlapping neighborhood graphs in panel (b) of Fig. 2.

3.3. Equilibrium selection: stable Nash equilibria

For any graph, there are potentially many Nash equilibria—ranging from specialized to distributed.[14] In this section, we consider stable equilibria. We use a simple notion of stability based on Nash tâtonnement, see e.g. [26]. We use this concept because it applies directly to games with continuous action spaces, and because it indeed reduces the number of equilibria in our setting in a natural way. Define $f_i(\mathbf{e})$ as the best-response of individual i to a profile $\mathbf{e}$ and define $\mathbf{f}$ as the collection of these individual best-responses $\mathbf{f} = (f_1, \ldots, f_n)$. An equilibrium $\mathbf{e}$ is *stable* if and only if there exists a positive number $\rho > 0$ such that for any vector $\boldsymbol{\epsilon}$ satisfying $\forall i, |\varepsilon_i| \leqslant \rho$ and $e_i + \varepsilon_i \geqslant 0$ the sequence $\mathbf{e}^{(n)}$ defined by $\mathbf{e}^{(0)} = \mathbf{e} + \boldsymbol{\epsilon}$ and $\mathbf{e}^{(n+1)} = \mathbf{f}(\mathbf{e}^{(n)})$ converges to $\mathbf{e}$.

This standard notion yields a strong result. Only specialized equilibria are stable. This result rests on the strategic substitutability of efforts of linked agents. Consider an equilibrium where everyone exerts some effort, and decrease the effort of an individual i by a small amount. His neighbor(s) will adjust by increasing their own efforts. This increase can lead i to reduce his effort even more. In this case, the initial equilibrium is not stable. This process does not work in specialized equilibria when every agent j who exerts no effort is linked to two specialists. If we reduce the effort of these specialists, agent j will not adjust. He has access to two sources of information, and a small reduction will not lead him to increase his own effort.

Stable profiles thus correspond to maximal independent sets of order 2. Given a graph $\mathbf{g}$, we show a stable equilibria exists if and only if there is a maximal independent set of order 2.

Theorem 2. *For any social structure* $\mathbf{g}$, *an equilibrium is stable if and only if it is specialized and every non-specialist is connected to* (*at least*) *two specialists. Hence*, *there exists a stable equilibrium in a graph* $\mathbf{g}$ *if and only if it has a maximal independent set of order* 2.

The following example illustrates.

Example 3. Stable equilibria. Consider the star graph with four agents in Fig. 1, and consider the Nash equilibrium where the center exerts e^* and peripheral agents exert no effort. The set of specialists is not a maximal independent set of order 2. In contrast, consider the equilibrium where all peripheral agents exert effort. This equilibrium is stable, as the set of specialists is a maximal independent set of order 2.

4. Welfare analysis

In this section we consider the welfare of different allocations of effort. We show that not only are specialized equilibria the only stable outcomes, they also can yield the highest welfare.

[14] We show in Section 6 that multiplicity of equilibria is robust to alternative specifications of the model.

4.1. Definition of welfare

To gain a basic understanding of welfare, we take a standard utilitarian approach. We specify social welfare of profile $\mathbf{e}$ for a graph $\mathbf{g}$ as the sum of the payoffs of the agents:

$$W(\mathbf{e};\mathbf{g}) = \sum_{i\in N} b\,(e_i + \overline{e}_i) - c\sum_{i\in N} e_i,$$

where recall $\overline{e}_i$ is the sum of the efforts of i's neighbors.

4.2. Efficient allocations

We say a profile $\mathbf{e}$ is *efficient* for a given structure $\mathbf{g}$ if and only if there is no other profile $\mathbf{e}'$ such that $W(\mathbf{e}';\mathbf{g}) > W(\mathbf{e};\mathbf{g})$. Since the welfare function is concave, in an efficient profile, for any individual i such that $e_i > 0$, we must have $\frac{\partial W(\mathbf{e};\mathbf{g})}{\partial e_i} = 0$; that is, for all $e_i > 0$

$$b'(e_i + \overline{e}_i) + \sum_{j\in N_i} b'(e_j + \overline{e}_j) = c, \tag{1}$$

where the left-hand side is the marginal social benefit from i's effort. In an efficient profile where an agent i does no effort ($e_i = 0$) we must have $\frac{\partial W(\mathbf{e};\mathbf{g})}{\partial e_i} \leqslant 0$.

We can characterize efficient profiles for some important graphs. Consider *regular graphs* (i.e., graphs where each player has the same number of neighbors k, and the number of neighbors is called the *degree* of the graph). On a regular graph of degree k, we find there is always an efficient profile where each agent does the same amount of effort e where e satisfies $b'((k+1)e) = \frac{c}{(k+1)}$. Each agent benefits from his own and his neighbors effort—hence each agent has benefits $b((k+1)e)$. And the marginal cost per individual is $\frac{c}{(k+1)}$. For example, in a circle with n agents, e satisfies $b'(3e) = \frac{c}{3}$. This allocation of effort solves the first-order conditions of welfare maximization. By concavity of welfare, it must be efficient.

In non-regular graphs, some agents might not contribute to the public good in the efficient allocation. Indeed, for a class of graphs that includes the star, we find that efficient profiles involve some agents exerting no effort. Consider any graph where one agent's neighborhood is a strict subset of another agent's neighborhood. In this case, the agent with smaller neighborhood should do no effort. For any graph $\mathbf{g}$ with two individuals i and j such that $i \cup N_i \subsetneq j \cup N_j$, $e_i = 0$ in any efficient profile. [15]

While these profiles maximize social welfare, individuals, acting non-cooperatively, will never choose these effort levels. No Nash equilibrium profile is efficient. We can see this outcome easily by comparing the efficiency condition (1) with the Nash equilibrium conditions above, where an individual considers only his own marginal benefits of effort and sets $b'(e_i + \overline{e}_i) = c$.

We illustrate the difference between efficient and equilibrium allocations in our next example.

Example 4. Efficient vs. equilibrium allocations. Consider the graph in Fig. 3, which represents two connected communities. Sociologists (e.g., [30,15]) have argued that links, or bridges, between communities increase opportunities for learning. Typically this literature does not consider

[15] Take a profile e such that $e_i > 0$. Define a new profile e' such that $e'_i = 0$, $e'_j = e_i + e_j$ and $e' = e$ otherwise. Then, $W(e') > W(e)$. Total costs are the same, but social benefits are strictly greater for e' since j's effort reaches more individuals in the population.

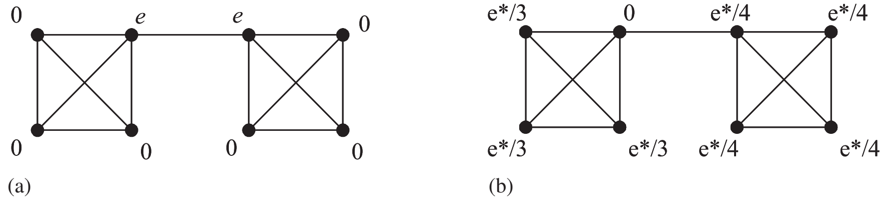

Fig. 3. Efficient vs. equilibrium allocations.

the incentives to generate new information. This example shows the negative effects of links when these incentives are important. In the efficient allocation, the agents who link the communities make all the contributions to the public good, as in (a). The neighborhoods of all other agents are subsets of these two agents' neighborhoods. Condition (1) implies that they both exert effort e such that $3b'(e) + 2b'(2e) = c$. The set of Nash equilibria, however, does not include this allocation. In equilibrium, at least one of the central agents does zero effort, as shown in panel (b), and, hence, the effort level of e^* must be distributed among the other agents in at least one community.

4.3. The "best" Nash equilibria

While no Nash equilibrium is efficient, here we ask which Nash equilibria yield highest welfare. We develop a method to compare the welfare of different equilibrium profiles. Recall, in any equilibrium, each agent has benefits of at least e^* effort. Hence, $nb(e^*)$ is the minimum aggregate benefits in any equilibrium. In equilibria where some agents do not exert effort but rely on specialists, these agents have the benefits of more than e^* effort. Their benefits are equal to $\sum_{j:e_j=0} [b(\bar{e}_j) - b(e^*)]$ where the summation is over all agents j who do not exert effort. We can therefore express the welfare of an equilibrium $\mathbf{e}$ as the sum of three terms:

$$W(\mathbf{e}; \mathbf{g}) = nb(e^*) + \sum_{j:e_j=0} [b(\bar{e}_j) - b(e^*)] - c \sum_i e_i, \qquad (2)$$

where the second term is the *benefit premium* that can arise from specialization.

In (2), we see a trade-off between benefit premia and effort costs. Distributed equilibria yield no benefit premia. Specialization yields benefit premia but at possibly higher cost. When the trade-off favors the benefit premia, specialization can yield higher welfare. The resolution of this trade-off depends on the benefits of information above e^*, embodied in the premium $\sum_{j:e_j=0}[b(\bar{e}_j)-b(e^*)]$. The size of this premium depends on the effort and location of specialists, and on the shape of the benefit function above e^*.

We can describe the shape of the benefit function as follows [16]: If b tends to be flat above e^*, benefit premia have close to zero value. Equilibria with the highest welfare are simply those with lowest total effort. The benefit premium is *largest* when b tends to be steep above e^* (that is, its slope does not fall much below c). Equilibria with high levels of total effort may then be beneficial; the benefit premium can outweigh the additional cost.

[16] Since we want to compare equilibria, and equilibria only depend on e^*, the different benefit functions we consider always satisfy $b'(e^*) = c$.

To understand this possibility more precisely, we introduce the following measure of the benefit function. Let

$$\sigma = \frac{b(ne^*) - b(e^*)}{c(n-1)e^*}.$$

Since b is increasing and concave, σ lies between 0 and 1. When σ is close to 1, the slope of benefit function above e^* does not fall by much; the slope of the benefit function is close to c. This measure derives solely from the benefit function and is independent on the graph. We can thus use it to compare equilibria in any given graph.

When σ is close to 1, we find that welfare simply depends on the sum of total effort that accrues to agents from others in the network. That is, welfare depends only on $\sum_i k_i e_i$, where, recall, k_i represents the number of neighbors of i. This quantity is a linear approximation of the difference between benefit premium and effort costs when σ is close to 1. For high σ, welfare is higher when agents gain much from their neighbors' efforts.

We thus have:

Proposition 1. *Consider a graph* $\mathbf{g}$ *and two Nash equilibria* $\mathbf{e}^1$ *and* $\mathbf{e}^2$ *on* $\mathbf{g}$. *There exists a* $\sigma_H < 1$ *such that for any benefit function satisfying* $b'(e^*) = c$ *and* $\sigma > \sigma_H$, $W(\mathbf{e}^1; \mathbf{g}) > W(\mathbf{e}^2; \mathbf{g})$ *if* $\sum_i k_i e_i^1 > \sum_i k_i e_i^2$.

To illustrate, we compare welfare of equilibria on a circle.

Example 5. Comparing welfare of Nash equilibria. Consider a circle with n agents, where n is even. Consider the specialized equilibria where alternate agents exert effort e^*, as shown for $n = 4$ in the right side of panel (c) in Fig. 1. We compare these equilibria to any other equilibrium for this graph. In the specialized equilibrium we specified, the sum $\sum_i k_i e_i$ is ne^*. In any other equilibrium, this sum is lower. For example, in distributed equilibria, this sum is $\frac{2ne^*}{3}$. Applying Proposition 1, we obtain that, if σ is high enough, the specialized equilibrium yields greater welfare.

Hence specialization can be beneficial in equilibrium. Since individual efforts are strategic substitutes, there is a limit to the level of total effort sustainable in equilibrium. When agents specialize, more effort is sustainable. When specialists are well-located in the graph, social benefits increase through the benefit premium. When σ is higher, the benefit premium outweighs the additional costs.

5. Positive and negative effects of new links

In the previous section, we considered which Nash equilibria yielded the highest overall welfare. In this section we ask how changes in graph itself affect welfare. We examine the welfare effects of adding a new link to a given graph. A new link has two, countervailing, effects. A link allows for greater access to the public good. But an agent with greater access also has less incentive to exert own effort. We show that this disincentive can lead to a loss in welfare.

We consider changes in the set of Nash equilibria when we add a new link. We could modify the analysis to consider changes in the set of stable equilibria, and similar insights obtain. We say an equilibrium profile $\mathbf{e}$ is *second-best* for a given graph $\mathbf{g}$ if and only if there is no other equilibrium profile $\mathbf{e}'$ such that $W(\mathbf{e}'; \mathbf{g}) > W(\mathbf{e}; \mathbf{g})$. Consider a graph $\mathbf{g}$ and two agents i and j who are not

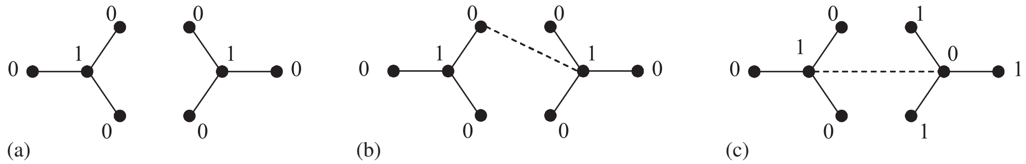

Fig. 4. Connecting two stars.

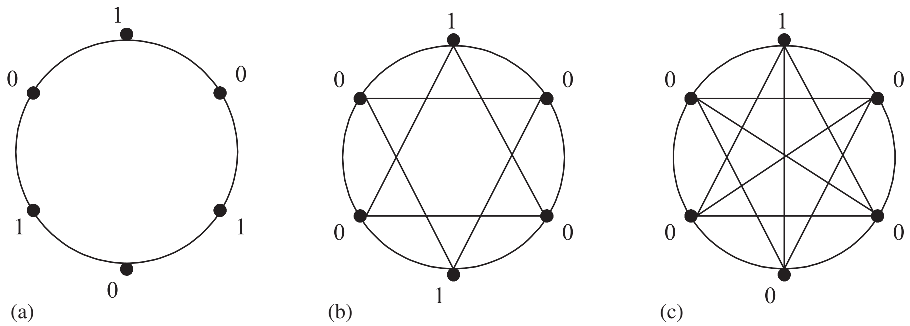

Fig. 5. Increasing integration.

linked in $\mathbf{g}$. Denote by $\mathbf{g} + ij$ the graph obtained by connecting i and j in $\mathbf{g}$. We say that the link leads to a "loss in welfare"when the second-best level of welfare for graph $\mathbf{g} + ij$ is lower than that for $\mathbf{g}$. Let $\mathbf{e}$ be a second-best equilibrium profile for the social structure $\mathbf{g}$. There are two cases. First, in $\mathbf{e}$, either i or j does not exert effort. In this case, $\mathbf{e}$ is also an equilibrium for $\mathbf{g}+ij$ and hence $W(\mathbf{e}; \mathbf{g} + ij) \geqslant W(\mathbf{e}; \mathbf{g})$. Second, in $\mathbf{e}$, both i and j exert effort. In this case, $\mathbf{e}$ is not an equilibrium for the structure $\mathbf{g} + ij$. Adding a link between the two destroys the equilibrium pattern. When effort by both agents is required to secure high aggregate benefits, the new link can lead to a loss in welfare.

Hence, a necessary condition for a loss in welfare from linking i and j is that both agents exert effort in all second-best equilibrium profiles on $\mathbf{g}$.

We illustrate the positive and negative effects of a new link in the following example.

Example 6. Positive and negative effects of new links. Consider the two stars in Fig. 4 panel (a). The figure shows the unique second-best equilibrium pattern—both centers are specialists. Connecting a peripheral agent to the center of the other star, Fig. 4 panel (b), does not disrupt the equilibrium. The link thus increases welfare. Connecting the two centers, Fig. 4 panel (c), destroys the equilibrium pattern and thus can decrease welfare. With the link between the two centers, the second-best equilibrium profiles involve the center of one star and peripheral agents of the other star as specialists. Welfare falls if the increased costs exceed the added benefit premium: $2ce^* > b(4e^*) - b(e^*)$. This is guaranteed if σ is low enough. In the local interactions graph in Fig. 5, we depict second-best equilibria when σ is high enough, so specialized equilibria are second-best. In (a), each agent has two neighbors, and alternate agents are specialists. In (b), each agent has four neighbors. Costs fall and the benefit premium increases, because one more agent has access to two sources of information. Welfare increases. The graph is complete in (c) and welfare falls. There can be at most one specialist; there is no possibility of a benefit premium. Hence, welfare can be higher when the graph is not complete.

6. Robustness of results

Our main insights carry over to three changes in the model's specifications: imperfect substitutes, convex costs, and agent heterogeneity. Specialization may emerge in equilibrium and has similar implications for welfare. In each case we extend Theorem 1, relating specialized equilibria to maximal independent sets in a graph.

Imperfect substitutability of efforts: First, we consider that an individual's own effort could be more beneficial to himself than to his neighbors. Let benefits equal $b(e_i + \delta \sum_{j \in N_i} e_j)$ where $0 < \delta \leqslant 1$ measures the extent to which own efforts and neighbor's efforts are substitutes. With this specification, we can apply Theorem 1 in [5] who study network games with quadratic utilities. [17] On any graph if δ is low enough, there exists a unique Nash equilibrium and in this equilibrium all agents do strictly positive effort. Using our language, there is a unique equilibrium and it is a distributed equilibrium. Hence, specialization disappears when δ is low enough; when efforts by one's neighbors have little value, individuals must exert some effort on their own. We show specialized equilibria exist for higher values of δ, where the precise value depends on the order of maximal independent sets in the graph. When agents have sufficiently many neighbors in a graph and the graph admits maximal independent sets of sufficiently high order, there exist specialized equilibria. Theorem 1 generalizes as follows.

Proposition 2. *Suppose an agent's benefits are $b(e_i + \delta \sum_{j \in N_i} e_j)$ where $0 < \delta \leqslant 1$. Let s be the smallest integer larger than or equal to $\frac{1}{\delta}$. Then, a specialized profile is an equilibrium if and only if the set of specialists is a maximal independent set of order s of the graph* **g**.

For instance, on the circle when n is even, there are maximal independent sets of order 2. Hence, there exist specialized equilibria for $\delta \geqslant \frac{1}{2}$. The equilibria involve alternate agents exerting e^*. On the star, there is a maximal independent set of order $n-1$. The profile where all agents at the periphery are specialists and the center exerts no effort is an equilibrium for $\delta \geqslant \frac{1}{n-1}$.

Specialization still can yield welfare benefits in this case. For example, welfare of specialized equilibria on the circle is $W = nb(e^*) + \frac{n}{2}[b(2\delta e^*) - b(e^*)] - c\frac{n}{2}e^*$. In the distributed equilibrium where everyone searches $\frac{1}{1+2\delta}e^*$, welfare is equal to $W = nb(e^*) - c\frac{n}{1+2\delta}e^*$. We see here the trade-off between the benefit premium from specialization and the increased search costs.

[17] With imperfect substitutability in our game, an individual i's best-reply function is given by

$$e_i = \max\left(0, e^* - \delta \sum_{j \neq i} g_{ij} e_j\right).$$

Ballester et al. [5] study a game with bilinear payoffs, where the best-reply function is

$$x_i = \max\left(0, \frac{\sum_{j \neq i} \sigma_{ij} x_j + \alpha_i}{-\sigma_{ii}}\right).$$

The best-reply functions coincide when $\alpha_i = e^*$, $\sigma_{ii} = -1$, and $\sigma_{ij} = -\delta g_{ij}$.

Convex costs: Suppose in our original game that effort costs $c(e_i)$ are increasing and convex and that $c'(0) > b'(+\infty)$. Convex costs would arise, for instance, when individuals allocate resources between public goods and private good consumption, as in [7]. [18] Convex costs drive the outcome towards effort sharing. In complete graphs, there is now a unique equilibrium where all individuals exert the same amount of effort. Specialization can still emerge in graphs that are not complete. Our equilibrium condition involves again maximal independent sets of higher orders. Theorem 1 extends as follows. Let effort level e^* still represent how much an isolated individual experiments. It now satisfies $b'(e^*) = c'(e^*)$.

Proposition 3. *Suppose that $c(e)$ is increasing and convex and that $c'(0) > b'(+\infty)$. Let s be the smallest integer such that $b'(se^*) \leqslant c'(0)$. Then, a specialized profile is a Nash equilibrium if and only if the set of specialists is a maximal independent set of order s of the graph* **g**.

As for welfare, specialization still generates a benefit premium. Although convexity of the cost function makes specialization less attractive, the benefit premium can still outweigh the higher effort costs.

Heterogeneous agents: Next, suppose that agents can derive different benefits from the public good and have different costs of effort. Let the benefit functions and cost functions be $b_i(e_i + \bar{e}_i)$ and $c_i e_i$. [19] We can use many of our techniques to analyze this case. Equilibrium outcomes can be represented by an idiosyncratic threshold effort level e_i^* such that in equilibrium, $e_i = 0$ when $\bar{e}_i \geqslant e_i^*$ and $e_i = e_i^* - \bar{e}_i$ otherwise. Individuals with higher benefits or lower costs have higher thresholds. On the complete graph, there is a (generically) unique equilibrium where the individual with highest threshold exerts all the effort. Heterogeneity thus leads to specialization. This finding is reinforced on graphs that are not complete. In a specialized equilibrium, the set of specialists must still be a maximal independent set of the graph. While this condition is not sufficient, existence is guaranteed on any graph.

Proposition 4. *Consider the heterogeneous agent model. All specialized Nash equilibria correspond to a maximal independent set of the graph. There always exists a specialized equilibrium where the agent with highest threshold is a specialist.*

In this heterogeneous agent case, the welfare comparison between different equilibria is, again, determined by a trade-off between the benefit premium and effort costs.

7. Conclusion

This paper introduces a network model of public goods. In this model, there is a fixed social structure, and agents choose how much to contribute to a public good when the good is non-excludable among their linked neighbors.

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[18] Suppose individuals allocate income y between effort e_i and private good consumption x_i. Assume separable utility: $u_i(\mathbf{e}, x_i) = b(e_i + \bar{e}_i) + a(x_i)$. With private good's price at 1, public good's price at p, the budget constraint is $x_i + pe_i = y$. The marginal cost of effort is then $pa'(y - pe_i)$ and is increasing in e_i.

[19] Heterogeneity can arise from a simple model of impure altruism. Suppose individual i obtains a 'warm glow' [1] of me_i for each neighbor that benefits from her experimentation e_i. This gain effectively lowers i's marginal cost of $c - k_i m$. Individuals with more links earn greater pleasure from generating new information and, hence, have lower effective costs of experimentation.

Our analysis could potentially guide empirical work. There is a growing field in economics studying innovation and diffusion of information, [20] and there are many studies that suggest that social structures affect experimentation and spread of information. [21] Our analysis suggests that individuals who have active social neighbors should have high benefits but exert little effort. We also expect individuals who have prominent social positions to bear less of the effort costs, and instead to rely on others' efforts. Thus our model indicates the importance of strategic and network efforts. For instance in the study of new crop adoption in developing countries, it would be important to investigate the strategic effects as in [24] in the context of network data of the type collected in [19].

Future theoretical research could investigate network formation and how information transmission feeds back into the evolution of social links. In our analysis, we look at existing networks; channels of information transmission are already in place when the need for new information appears. Thus, our setting is probably more appropriate to understand short-run patterns of experimentation and information diffusion within established networks. Looking at the long-term formation and evolution of networks is a promising avenue for future research.

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Appendix

Proof of Theorem 1. See Proposition 2 with $\delta = 1$. $\square$

Proof of Theorem 2. Recall, $\mathbf{f}(\mathbf{e})$ denotes best-responses to profile $\mathbf{e}$, where $f_i(\mathbf{e}) = \max(e^* - \bar{e}_i, 0)$. Our proof relies on the following lemma.

[20] There is a growing empirical interest in social learning, and a new set of papers attempts to identify network effects on technology adoption [6,19,35].

[21] Foster and Rosenzweig [24] find evidence that information is a public good and that people free ride; the level of experimentation is less than is socially optimal. In industrial organization, [33] provides evidence that research and knowledge spillovers are local, and hence, provides support for our assumption that information is a public good along geographic and social links. See, also for example, [28] for a descriptive account. People who have extensive knowledge of the marketplace are usually quite peripheral. Rogers [36] reports similar findings when contrasting the social position of "innovators," who typically are the first to experiment, to that of "early adopters," who rely on innovators' experiences before making their decisions: "early adopters are a more integrated part of the local social system than are innovators," (p. 263).

Lemma A1. *If* $\mathbf{e} \leqslant \mathbf{e}'$, *then* $\mathbf{f} \circ \mathbf{f}(\mathbf{e}) \leqslant \mathbf{f} \circ \mathbf{f}(\mathbf{e}')$.

Proof. Suppose that $\forall i$, $e_i \leqslant e_i'$. Then, $e^* - \bar{e}_i \geqslant e^* - \bar{e}_i'$, hence $\max(e^* - \bar{e}_i, 0) \geqslant \max(e^* - \bar{e}_i', 0)$ and $\mathbf{f}(\mathbf{e}) \geqslant \mathbf{f}(\mathbf{e}')$. Applying $\mathbf{f}$ again to this inequality yields the result. □

Consider first an equilibrium that is not specialized and denote by $J = \{j : 0 < e_j < e^*\}$. Let $\rho > 0$ be a small number and define a perturbation $\boldsymbol{\epsilon}$ as follows: $\forall j \in J$, $\varepsilon_j = \rho$ and $\forall j \notin J$, $\varepsilon_j = 0$. We wish to show that $\mathbf{f} \circ \mathbf{f}(\mathbf{e} + \boldsymbol{\epsilon}) \geqslant \mathbf{e} + \boldsymbol{\epsilon}$. There are three cases. (1) If i is such that $e_i = 0$, then $\varepsilon_i = 0$, $f_i(\mathbf{e} + \boldsymbol{\epsilon}) = 0$ and $f_i(\mathbf{f}(\mathbf{e} + \boldsymbol{\epsilon})) \geqslant 0 = e_i + \varepsilon_i$. (2) If i is such that $e_i = e^*$, then $\varepsilon_i = 0$ and $\forall j \in N_i$, $e_j = \varepsilon_j = 0$. This yields $f_i(\mathbf{e} + \boldsymbol{\epsilon}) = f_i(\mathbf{f}(\mathbf{e} + \boldsymbol{\epsilon})) = e^* = e_i + \varepsilon_i$. (3) Finally, suppose that $i \in J$. If ρ is small enough, we have $\forall j \in J$, $\bar{\varepsilon}_j \leqslant e_j$, hence $f_i(\mathbf{e} + \boldsymbol{\epsilon}) = e_i - \bar{\varepsilon}_i$. Then, $f_i(\mathbf{f}(\mathbf{e} + \boldsymbol{\epsilon})) = e_i + \sum_{j \in J \cap N_i} \bar{\varepsilon}_j$. Since i has at least one neighbor in J, $\sum_{j \in J \cap N_i} \bar{\varepsilon}_j \geqslant \rho$ hence $f_i(\mathbf{f}(\mathbf{e} + \boldsymbol{\epsilon})) \geqslant e_i + \varepsilon_i$. Therefore, $\mathbf{f} \circ \mathbf{f}(\mathbf{e} + \boldsymbol{\epsilon}) \geqslant \mathbf{e} + \boldsymbol{\epsilon}$. By applying the lemma we can see that for any finite number k, $\mathbf{f}^{(2k)}(\mathbf{e} + \boldsymbol{\epsilon}) \geqslant \mathbf{e} + \boldsymbol{\epsilon}$, which is strictly greater than $\mathbf{e}$. Therefore, the sequence of best-responses never converges back to $\mathbf{e}$ and the equilibrium is not stable.

Consider next a specialized equilibrium $\mathbf{e}$ such that i is a non-specialist who is connected to a unique specialist j. Let $\rho > 0$ be a small number and define a perturbation $\boldsymbol{\epsilon}$ as follows: $\varepsilon_i = \rho$ and $\varepsilon_l = 0$ if $l \neq i$. Then, clearly, $e_l^{(1)} = e_l$ for any l except j and $e_j^{(1)} = e^* - \rho$. Next, $e_l^{(2)} = e_l$ for any l except for neighbors of j whose only specialist neighbor is j. These agents, which include i, all play ρ. This means that $\mathbf{f} \circ \mathbf{f}(\mathbf{e} + \boldsymbol{\epsilon}) \geqslant \mathbf{e} + \boldsymbol{\epsilon}$ and we can apply the same argument as above, hence this equilibrium is not stable.

Finally, let us prove that specialized equilibria in which every non-specialist is connected to (at least) two specialists are stable. Take $\mathbf{e}$ such an equilibrium, let I be the set of specialists in $\mathbf{e}$, and let $\delta = \frac{1}{n^2} e^*$. Consider any perturbation $\boldsymbol{\epsilon}$ such that $\forall i$, $|\varepsilon_i| < \delta$ and $\varepsilon_i + e_i \geqslant 0$. First, determine $\mathbf{e}^{(1)}$. For $i \notin I$, $\bar{e}_i^{(0)} = \bar{e}_i + \bar{\varepsilon}_i = |N_i \cap I| e^* + \bar{\varepsilon}_i$. Since $|\bar{\varepsilon}_i| \leqslant n\delta \leqslant e^*$, this implies that $\bar{e}_i^{(0)} \geqslant e^*$, hence $e_i^{(1)} = 0$. Despite the perturbation, non-specialists still do no effort. For $i \in I$, $\bar{e}_i^{(0)} = \bar{\varepsilon}_i$, hence $e_i^{(1)} = e^* - \bar{\varepsilon}_i$. Next, determine $\mathbf{e}^{(2)}$. For $i \notin I$, $\bar{e}_i^{(1)} = |N_i \cap I| e^* - \sum_{j \in N_i \cap I} \bar{\varepsilon}_j$. Since $|\sum_{j \in N_i \cap I} \bar{\varepsilon}_j| \leqslant n^2 \delta \leqslant e^*$, we obtain again that $e_i^{(2)} = 0$. Finally, for $i \notin I$, $\bar{e}_i^{(1)} = 0$ hence $e_i^{(2)} = e^*$. We showed that $\mathbf{f} \circ \mathbf{f}(\mathbf{e} + \boldsymbol{\epsilon}) = \mathbf{e}$, hence $\mathbf{f}^{(k)}(\mathbf{e} + \boldsymbol{\epsilon}) = \mathbf{e}$ for all $k \geqslant 2$, hence the equilibrium is stable. □

Proof of Proposition 1. Consider an equilibrium $\mathbf{e}$ and i such that $e_i = 0$. Since b is increasing and concave, we have $\sigma c(\bar{e}_i - e^*) \leqslant b(\bar{e}_i) - b(e^*) \leqslant c(\bar{e}_i - e^*)$. This means that

$$\sigma c \left[\sum_{i : e_i = 0} (\bar{e}_i - e^*) - \sum_i e_i \right] + (1 - \sigma) c \sum_i e_i \leqslant W(\mathbf{e}, \mathbf{g}) - n b(e^*)$$

$$\leqslant c \left[\sum_{i : e_i = 0} (\bar{e}_i - e^*) - \sum_i e_i \right].$$

In addition, $\sum_{i : e_i = 0} (\bar{e}_i - e^*) = \sum_{i \in N} (e_i + \bar{e}_i) - n e^*$. By switching the double summation, we obtain $\sum_{i \in N} (e_i + \bar{e}_i) = \sum_i \sum_j g_{ij} e_j = \sum_j \sum_i g_{ij} e_j = \sum_j (k_j + 1) e_j$. Substituting yields

$$\sigma c \sum_j k_j e_j + (1 - \sigma) c \sum_i e_i \leqslant W(\mathbf{e}, \mathbf{g}) - n[b(e^*) - c e^*] \leqslant c \sum_j k_j e_j.$$

Hence, as σ tends to 1, $W(\mathbf{e}, G) - n[b(e^*) - ce^*]$ tends to $c\sum_j k_j e_j$. (We consider benefit functions for which $b'(e^*) = c$ so that equilibria are not affected by changes in σ). Therefore, there exists a threshold $\sigma_H < 1$ such that if $\sigma > \sigma_H$, $\sum_j k_j e_j^1 > \sum_j k_j e_j^2$ implies that $W(\mathbf{e}^1, \mathbf{g}) > W(\mathbf{e}^2, \mathbf{g})$. □

Proof of Proposition 2. Consider a specialized equilibrium where I is the set of specialists. Specialists play a best-response if all their neighbors exert zero effort. This means that I is an independent set of the graph. A non-specialist i plays a best-response if $\delta\sum_{j\in N_i} e_j \geqslant e^* \Leftrightarrow |N_i \cap I| \geqslant \frac{1}{\delta} \Leftrightarrow |N_i \cap I| \geqslant s$. This means that all agents not in I are connected to at least s agents in I. Combining both properties yields the result. □

Proof of Proposition 3. With convex experimentation costs $c(e_i)$, individual i's best-response is to play 0 if $b'(\bar{e}_i) \leqslant c'(0)$ and to play e_i such that $b'(e_i + \bar{e}_i) = c'(e_i)$ otherwise. On the complete graph, there is a unique equilibrium where the effort of any individual is e such that $b'(ne) = c'(e)$. In general graphs, consider a specialized equilibrium where I is the set of specialists. Specialists play a best-response if all their neighbors exert zero effort. A non-specialists i play a best-response if $b'(\sum_{j\in N_i} e_j) \leqslant c'(0) \Leftrightarrow b'(|N_i \cap I|e^*) \leqslant c'(0) \Leftrightarrow |N_i \cap I| \geqslant s$. This yields the result. □

Proof of Proposition 4. Order agents through decreasing thresholds $e_1^* \geqslant \cdots \geqslant e_n^*$. Construct a maximal independent set I as follows. Include individual 1 in I. Then remove 1 and all her neighbors. Add to I the remaining agent with highest threshold. Then, remove her neighbors and repeat the operation till no agent is left. The profile where each agent i in I searches e_i^* while others do no search is a specialized equilibrium. □

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