

SOCIAL STATUS IN NETWORKS

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ABSTRACT. We study social comparisons and status seeking in an interconnected society. Individuals take costly actions that have direct benefits and also confer social status. A new measure of interconnectedness—cohesion—captures the intensity of incentives for seeking status. Equilibria stratify players into social classes, with each class’s action pinned down by cohesion. A network decomposition algorithm characterizes the highest (and most inefficient) equilibrium. Members of the largest maximally cohesive set form the highest class. Alternatively, players not belonging to sets more cohesive than the set of all nodes constitute the lowest class. Intermediate classes are identified via iterated application of a cohesion operator. We characterize the networks that accommodate multiple-class equilibria.

Keywords: networks, status, social classes, strata, cohesion, class equilibrium, inefficiency, supermodular games.

Since at least Veblen’s (1899) classic study on conspicuous consumption, economists and social scientists have recognized that social comparisons influence individual decisions and welfare. Recent empirical studies establish that “happiness” depends on relative rather than absolute levels of wealth or consumption, especially within one’s own community.¹ People compare their income, their consumption, their assets, their effort or output at work, and their performance in school to those of people around them, and they strive to maintain their position among family, friends, neighbors, coworkers, and peers with similar demographics.²

This paper studies social comparisons and status seeking in an interconnected society. Players are embedded in a social network, and each player’s neighbors constitute his individual reference group. Since players’ social contacts can overlap, players are indirectly connected to many others in society, and actions distant in the network can spill over through

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¹For a review of this literature see Clark et al. (2008).

²Veblen (1899), Merton (1938), and Duesenberry (1949) provide classic discussions of conspicuous consumption and social comparisons. More recent work by Frank (1985a, 1999, 2007) provides evidence from economics, sociology, and psychology on the implications of status considerations for salaries, workplace incentives, consumption of luxury goods, savings, prices, happiness, health, laws, inefficiency, and inequality.

the links. In this context, we study a simultaneous move game in which actions not only have intrinsic costs and benefits but also confer status. Specifically, we assume that individuals lose status when they are lower on the totem pole in terms of their actions and that the status loss is more pronounced when their actions are further below those of higher-ranked neighbors. The personal intensity of status losses is captured by individual-specific parameters. The analysis uncovers how these parameters and the network structure shape equilibrium actions and the formation of social strata.

Our main finding is that a novel measure of interconnectedness, which we call *cohesion*, determines equilibrium outcomes. Cohesion captures both the number of links and the importance of links for status comparisons within a subset of nodes.³ The more cohesive a set is, the more its members compare themselves to other players within the set, who are also comparing themselves to one another, leading to a form of “rat race,” which in equilibrium may yield a high level of status seeking activity. New links between players can positively or negatively impact the cohesion of different sets, which can then increase or decrease overall equilibrium activity. Thus adding connections between different segments of society could diffuse or exacerbate the losses from status seeking.

The analysis indicates that the cohesion of subsets of society is a driver of spending and thus yields predictions for spending patterns, including conspicuous consumption and charitable contributions. In our model, cohesion derives from three sources: agents’ intrinsic value from spending, social links among agents, and the weights agents place on status comparisons with neighbors in the social network. Increases in the parameters capturing intrinsic values and status comparisons, as well as changes in social connections that increase cohesion, generate more spending. Such comparative statics might explain, for example, trends in charitable contributions. Consider a set of people, e.g., wealthy individuals, who value charitable giving and for whom contributions confer status. As this set of donors becomes more interconnected, through friendships per se or through media comparisons, cohesion increases. A small number of individuals with high intrinsic motivation to give can amplify their peers’ desire to contribute, inducing them to donate more than they would in the absence of any social pressure.

The Giving Pledge (<http://givingpledge.org>)—“a commitment by the world’s wealthiest individuals and families to dedicate the majority of their wealth to philanthropy”—could be one such instance, which mirrors activities in charitable giving more generally. This movement, started by Bill and Melinda Gates and Warren Buffet, encourages the world’s wealthiest individuals (“billionaires,” according to the website) to pledge to give away more

³Morris (2000) finds that a version of cohesion plays a central role in the analysis of a model of contagion in networks. In that context, the cohesion of a set of nodes is defined as the minimum *proportion* of links each node has within the set, whereas in our setting the cohesion of a set is the minimum *number* of (weighted) links each node has within the set. See Section A for a formal definition.

than half of their wealth. The website documents the motivations and intentions of those who join the cause with photos, profiles, and pledge letters. The Pledge virtually connects this set of people to one other. Lists of donor names and pledges are ubiquitous in fundraising campaigns, small and large, along with receptions, dinners, ball games, and other events where donors meet face-to-face. While such lists and events could encourage giving through several channels (e.g., gift exchange, sense of community), our analysis pinpoints a critical feature of the network structure—cohesion—that enhances philanthropy.

Our research departs from previous theoretical work on status seeking by considering—at the same time—“local” status concerns and “upward looking” comparisons. Modeling local status concerns naturally calls for a network formulation. In contrast, the first models of status seeking involve anonymous individuals in a large population. In Frank’s (1985b) original model, individuals care about both their own consumption and status, where the latter is defined by an individual’s position in the distribution of consumption in the population. Building on Frank (1985b), Hopkins and Kornienko (2004) analyze the equilibria of a status seeking game with a utility function that is multiplicative in consumption and status. Due to a type of rat race, status concerns lead to lower utility at every income level, even as incomes increase. In both models, players overspend on conspicuous consumption relative to the social optimum. Moreover, a player’s status in equilibrium is determined exactly by his position in the initial income distribution. Hence excessive equilibrium consumption has no actual effect on a player’s standing in the social hierarchy.

In line with our model, much empirical work supports the hypothesis that status comparisons are local; an individual’s satisfaction with his standing in the social hierarchy is based on his reference group. Evidence suggests that the composition of reference groups is partly driven by demographics.⁴ Luttmer (2005) finds that an individual’s self-reported happiness declines in the earnings of neighbors with similar educational attainment but is relatively insensitive to the earnings of neighbors with different levels of education. Reference groups may include large social circles, but can also be small, as in Neumark and Postlewaite (1998), who show that the income of a woman’s in-laws is predictive of whether she enters the labor market. While some research suggests that people may choose their reference group, we consider a snapshot in time when the network is formed and exogenously given. It is clear that exogenous exposure to various social environments is an important force driving status concerns and “spontaneous comparisons” are often unavoidable (Crosby (1976), Mussweiler and Bodenhausen (2002)).

Prior theoretical work that incorporates local status concerns assumes that players compare themselves to both their inferior and their superior neighbors. Ghiglini and Goyal

⁴See Festinger (1954), Frank (1985b), Van de Stadt et al. (1985), Luttmer (2005), and Kuhn et al. (2011).

(2010) build a network model in which each player’s utility depends on the difference between his consumption and the average neighborhood consumption. They find that each player’s consumption is proportional to his centrality in the network. In general, centrality does not cluster players into classes with similar actions, so Ghiglino and Goyal do not obtain the natural class stratification that our framework delivers. In our model, players compare themselves only to their superiors: they consider their rank and look up, rather than down, when comparing their actions to those of neighbors. This hypothesis is articulated in the classic works of Veblen (1899) and Duesenberry (1949) and also echoed in the modern study of Frank (1985a). Loewenstein et al. (1989) provide support for the hypothesis in an experiment on dispute resolution where subjects were concerned more with disadvantageous than with advantageous inequality. Some empirical evidence for the hypothesis can be found in the study of Ferrer-i-Carbonell (2005), which shows that self-reported life satisfaction of West Germans is affected by higher but not lower income individuals in reference groups. We borrow the exact formulation of status concerns from Stark and Wang (2005), who also review the literature on “relative deprivation,” which validates the upward comparisons specification.

Our preliminary results concern the welfare losses that arise from status seeking. We show that the status game is supermodular, which then implies that the set of Nash equilibria forms a lattice and delivers comparative statics with respect to payoff parameters. All players are worse off in equilibria that exhibit higher status seeking activity, and welfare in the maximum and minimum equilibria is decreasing with respect to the status concern parameters. Increases in status concerns lead to higher equilibrium actions. Thus our analysis upholds—in a network setting—previous results and contentions that status concerns can lead to distorted status seeking patterns (cf. Frank (1985b), Hopkins and Kornienko (2004), and Ghiglino and Goyal (2010)). When players have identical direct costs and benefits for actions, there exists an equilibrium in which all players take the same low action, equivalent to the optimal action in the absence of any status concerns. In all other equilibria players “run in place to stay ahead.” They take costly high actions in order to maintain their status and reduce their utility loss from being outranked by their neighbors. Increased status seeking activity is thus “wasteful,” in that it generates Pareto inferior equilibria. However, we should point out that our welfare comparative statics do not extend to a setting in which actions also have positive externalities in addition to the assumed negative status externalities.

The substantive results of our research relate the shape of equilibria to the cohesion of sets in the network. Equilibria are characterized by a partition of the players into *status classes* such that all players forming each class take an identical action. This action is sufficiently low so that all members have incentives to keep up with their class and sufficiently high so that no one wants to move into a higher class. The former bound depends on the cohesion

of the players who attain a status at least as high as the given class; the latter depends on the cohesion of the players who have strictly higher status with the addition of any single player from the class under consideration.

Since the maximum equilibrium exhibits the highest, and most inefficient, level of status seeking, we pursue the characterization of this equilibrium for our third set of results. The characterization of equilibrium social strata suggests that players who belong to more cohesive segments of society have more incentives to strive for status. Based on this intuition, we show that status classes in the maximum equilibrium can be determined using the following top-down procedure. The highest status class consists of the union of all sets of players that have maximum cohesion. Then the second highest class is formed by the remaining players who belong to any maximally cohesive superset of the highest class, and so on. Actions for each class are pinned down by the cohesion of the set of players who achieve at least the same status as the class. Alternatively, we construct a bottom-up characterization of the maximum equilibrium. The lowest status class is formed by those players who do not belong to any set that is more cohesive than the entire set of nodes, the next lowest class consists of the remaining players who are not part of sets that are more cohesive than the set which excludes the lowest class, and so on. While both the top-down and the bottom-up characterizations involve the inspection of a collection of sets that grows exponentially in the number of players, we are able to develop an algorithm that computes the maximum equilibrium in polynomial time. We also establish that each player's action in the maximum equilibrium is given by the highest cohesion achieved by a set that includes him.

The existence of *class equilibria*, in which players are segregated in several social classes, also depends on cohesion. We establish that there exists a class equilibrium if and only if there is a strict subset of players whose cohesion falls whenever a single player is added. Such a group of players serves as the highest status class in an equilibrium, and the condition above guarantees that no outside player has incentives to emulate it.

The paper makes two main technical advances. First, it provides an understanding of the key role that cohesion plays in the emergence of social strata. Second, it employs the concept of cohesion to characterize the maximum equilibrium via a decomposition of the network into social status classes. The decomposition features a complex combinatorial structure, demonstrated by the alternative top-down and bottom-up derivations of the maximum equilibrium as well as by the underlying polynomial-time algorithm. The graph theoretical techniques we develop could prove useful in other settings where networks can be parsed using similar criteria.

The rest of the paper is organized as follows. The next section introduces the social status game. Section II establishes the lattice structure of the equilibrium set and highlights the inefficiencies of equilibria with high status seeking activity; it also provides comparative

statics with respect to the benefit and status parameters. In Section III, we define cohesion and show how the cohesion of different sets in the network underlies the status classes that emerge in equilibrium. Section IV constructs two algorithms that identify the maximum action equilibrium: one starts from the highest status class and works down, the other starts at the lowest class and works up. Multiple-class equilibria are studied in Section V, and concluding remarks are provided in Section VI.

I. THE SOCIAL STATUS GAME

A finite set N of *players* participates in the following *social status game*. Each player $i \in N$ simultaneously picks an action $a_i \in [0, \infty)$ which has private benefits and costs as well as social status implications. Let $a = (a_i)_{i \in N} \in [0, \infty)^N$ denote the realized action profile; as usual, a_{-i} denotes the action profile of players other than i . Actions may reflect the consumption level for a luxury good, the size of the contribution to a public good, or the amount of effort expended in a contest. The *payoff function* $u_i : [0, \infty)^N \rightarrow \mathbb{R}$ of player $i \in N$ is specified by

$$u_i(a) = \alpha_i a_i - \frac{a_i^2}{2} - \sum_{j \in N} \beta_{ij} \max(a_j - a_i, 0),$$

where $\alpha_i \geq 0$ and $(\beta_{ij} \geq 0)_{j \in N}$ represent the *benefit* and *status* parameters, respectively, of player i .⁵ The first two terms capture standard linear benefits, $\alpha_i a_i$, and quadratic costs, $a_i^2/2$, of action a_i for player i . In the case of a car purchase, α_i would quantify the individual utility from the car; for charitable contributions, α_i could represent the “warm glow” (Andreoni (1990)) from the donation. The *status loss term* $\sum_{j \in N} \beta_{ij} \max(a_j - a_i, 0)$ captures i ’s status concerns; player i experiences a disutility when her action is lower than that of her neighbors and β_{ij} represents the weight that player i places on a particular player j . This specification of status losses was introduced by Stark and Wang (2005) in the context of a model of relative deprivation. In the introduction, we provided empirical support for the “local” (player i ’s reference group for status comparisons includes only players j such that $\beta_{ij} > 0$) and “upward looking” (player i suffers status losses only relative to players j with $a_j > a_i$) nature of status concerns embedded in our model.

Let α denote the vector of direct benefits $(\alpha_i)_{i \in N}$, β the matrix of social comparison weights $(\beta_{ij})_{i,j \in N}$, and u the corresponding collection of payoff functions $(u_i)_{i \in N}$. We refer to the strategic form game defined above as the *status game* with parameters (α, β) .

The matrix β is interpreted as a *weighted network*; β_{ij} is the weight of the directed *link* ij .⁶ Alternatively, the social comparison weights β may be derived from an *unweighted (directed) network* $g = (g_{ij})_{i,j \in N}$, in which $g_{ij} \in \{0, 1\}$ for all $i, j \in N$ and $g_{ij} = 1$ is interpreted as

⁵For notational convenience, define $\beta_{ii} = 0$.

⁶We allow for the possibility that $\beta_{ij} \neq \beta_{ji}$.

the existence of a social link from player i to player j ($g_{ii} = 0$ for all $i \in N$).⁷ In this case, $N_i = \{j \in N \mid g_{ij} = 1\}$ constitutes player i 's *reference group* or *neighbors*. We can take an unweighted network g as a primitive of the model describing the structure of social connections and allow for any specification of status parameters β such that $\beta_{ij} > 0$ if and only if $g_{ij} = 1$. In examples, we consider two salient profiles of status parameters β derived from an unweighted network g . Under *aggregate status concerns*, we simply set $\beta = g$. Alternatively, analogous to peer effect models, we can assume that each neighbor has the same normalized weight, $\beta_{ij} = g_{ij}/(|N_i| + 1)$ for all $i, j \in N$.⁸ In this latter specification, which we call *normalized status concerns*, the number of individuals in player i 's reference group has little effect on her status loss per se; it is the fraction of i 's neighbors in g (from the “inclusive” neighborhood $N_i \cup \{i\}$) who outrank her that factors into her status loss. To take an example from charitable giving, aggregate status concerns emphasize the possibility that an individual could feel worse being the lowest contributor out of one hundred acquaintances versus the lowest contributor out of ten. While we employ aggregate and normalized status concerns in examples and illustrations, it should be emphasized that our results hold for general status parameters.

Mathematically, the status loss term features both ordinal and cardinal aspects of social comparison. A player's loss of social status depends on the set of neighbors who outrank him as well as the degree by which they do so. This is an important aspect of interpersonal comparisons. For example, assume that people compare themselves with others on the basis of the brand of their cars. Imagine an individual who owns a Toyota (inexpensive car), and consider the following scenarios:

- (1) Most of the individual's neighbors also own a Toyota but one owns a BMW (medium-priced car).
- (2) Most neighbors own a Toyota but one owns a Ferrari (expensive car).
- (3) All of the individual's neighbors own BMWs.

In a model of purely ordinal social comparisons, the individual would experience the same status satisfaction in scenarios (1) and (2). This seems implausible because the individual might feel particularly unsuccessful compared to the Ferrari owner. In a cardinal world where the individual compares the quality of his car to the average quality of neighbors' cars, scenarios (2) and (3) may be identical. In particular, purchasing a BMW would have an identical effect on the individual's status in both scenarios. It seems reasonable that the individual would have more incentives to upgrade his car in scenario (3) because he needs to catch up with his entire neighborhood, rather than a single high-status neighbor. Our simple model of social comparisons can distinguish between the three cases.

⁷We say that the network g is *undirected* if $g_{ij} = g_{ji}$ for all $i, j \in N$.

⁸In both specifications, we can scale the status losses by a constant factor relative to the quadratic costs.

A. Extensions of the Model. The model and the underlying results can be easily generalized in two distinct directions. First, we can allow for more general action costs than the quadratic specification in the benchmark model. Indeed, all the results extend to a setting in which each player i incurs a cost $f_i(a_i)$ for taking action a_i , where f_i is a differentiable function such that $f_i'(0) = 0$ and f_i' is continuous, strictly increasing, and unbounded.

A second relevant generalization of the model incorporates payoff externalities that are orthogonal to strategic considerations (unlike the assumed status externalities). These externalities can be represented by adding a term of the form $e_i(a_{-i})$ (or $e_i(a_{N_i})$ for local externalities) to player i 's payoff function. Such externalities clearly do not affect any player's strategic decisions and leave the set of equilibrium action profiles unchanged. This means that the characterizations of equilibrium action levels and status classes from Sections III-V apply to the case with externalities verbatim. However, the welfare comparative statics in the next section (Propositions 3 and 4) do not extend to the setting with externalities.

II. PRELIMINARY RESULTS AND COMPARATIVE STATICS

We study the pure strategy Nash equilibria of this game which we simply refer to as *equilibria* henceforth. We first characterize best responses and demonstrate that the game is supermodular. Due to the additive and linear features of pairwise comparisons embodied in the status loss term, a player's marginal status loss is a step function in her action and her best response is isotone with respect to other players' actions. With this structure, an intuitive formulation of the best response dynamics yields the minimum and the maximum equilibria. We show that higher-action equilibria are Pareto-dominated by lower-action equilibria. We further conduct comparative statics on utility function parameters, finding that higher status concerns lead to equilibria with higher actions and lower utility for all. After establishing these characteristics of the equilibrium set, we turn to network features in Section III.

A. Best Responses. To gain some intuition for the structure of best responses, note that player i must weigh the marginal cost of action a_i , which is simply a_i , against the marginal private benefit α_i and the marginal effect on his status. We can rewrite i 's status loss as follows

$$\sum_{\{j \in N \mid a_j > a_i\}} \beta_{ij}(a_j - a_i).$$

For any action profile a_{-i} of i 's opponents, the expression above is piecewise linear in a_i , with kinks at the finite set of points $\{a_j \mid j \in N\}$. A marginal increase in i 's action at a point a_i that is not a kink creates a marginal improvement in i 's status of $\sum_{\{j \in N \mid a_j > a_i\}} \beta_{ij}$. Thus the marginal incentives for i to reduce her status loss are summarized by the following

function of the action profile:

$$r_i(a) = \sum_{\{j \in N \mid a_j > a_i\}} \beta_{ij}.$$

The first preliminary result formalizes the intuition that a best response needs to balance the marginal cost of higher actions with the marginal gain in direct benefits and the marginal reduction in status loss. The proof of this and all other results of the paper can be found in the Appendix.

Lemma 1. *Each player i has a unique best response to any pure action profile of her opponents a_{-i} , which is given by*

$$B_i(a_{-i}) := \min\{a_i \mid a_i \geq \alpha_i + r_i(a_i, a_{-i})\}.$$

Let $B : [0, \infty)^N \rightarrow [0, \infty)^N$ denote the *best response function* defined by $B(a) = (B_i(a_{-i}))_{i \in N}$. For any action profile a , consider the sequence $(B^t(a))_{t \geq 0}$ obtained by iterating the best response function starting at the action profile a . We call this sequence the *best response dynamics* initiated at a , and we say that the best response dynamics initiated at a *converges in finite time* to an action profile \tilde{a} if there exists $t \geq 0$ such that $B^t(a) = B^{t+1}(a) = \tilde{a}$. The latter condition implies that \tilde{a} is an equilibrium of the status game. Our second preliminary result shows that the best response dynamics takes a finite number of values.

Lemma 2. *For any action profile a , the best response dynamics initiated at a takes a finite number of values.*

To analyze the convergence of the best response dynamics and its relation to Nash equilibria, we introduce the following notation and definitions for orders and functions. Let \succeq denote the partial order on \mathbb{R}^N defined by

$$\forall x, y \in \mathbb{R}^N : x \succeq y \iff x_i \geq y_i, \forall i \in N.$$

Note that \mathbb{R}^N endowed with the order \succeq constitutes a lattice. A function $f : C \rightarrow D$ with $C, D \subseteq \mathbb{R}^N$ is *isotone* if $f(x) \succeq f(y)$ for any $x, y \in C$ with $x \succeq y$. Similarly, $f : C \rightarrow D$ with $C \subseteq \mathbb{R}, D \subseteq \mathbb{R}^N$ is isotone if $f(x) \succeq f(y)$ for any $x \geq y$ in C . Note that the generic variables $a, u(a), \alpha$ can be viewed as elements of the lattice (\mathbb{R}^N, \succeq) . It follows from Lemma 1 that the best response function B is isotone.

The following corollary follows from the observation that whenever a and $B(a)$ are ranked in the order \succeq , the fact that B is isotone implies that the best response dynamics initiated at a forms a chain in the lattice (\mathbb{R}^N, \succeq) . By Lemma 2, the dynamics takes only a finite set of values, so the chain must become constant in a finite number of steps.

Corollary 1. *For any action profile a such that $a \succeq B(a)$ ($B(a) \succeq a$), the best response dynamics initiated at a converges in finite time to an equilibrium of the status game that is dominated by (dominates) a in the order \succeq .*

B. Lattice Structure and Inefficiencies of Equilibria. In this section we establish that the equilibrium set has a lattice structure and relate this structure to inefficiencies due to status-seeking. The equilibrium set is bounded by an equilibrium with the lowest actions and one with the highest actions. We show that higher-action equilibria are Pareto dominated. Striving for status leads to a rat race in which everyone overspends to maintain their standing, in many cases leading to higher costs but no status gains. Conducting comparative statics, we further show that increases in any status concern parameter β_{ij} (weakly) lower all individuals' utilities. When player i cares more about her standing relative to j , she increases her action, leading to higher overall actions, and costs again outweigh any potential status benefits.

The first result demonstrates that the set of equilibria has a lattice structure. The proof establishes that the status game is supermodular and then simply applies the results of Milgrom and Roberts (1990).

Proposition 1. *The set of equilibria of the status game with parameters (α, β) is a sublattice of (\mathbb{R}^N, \succeq) . There exists a minimum equilibrium \underline{a} and a maximum equilibrium \bar{a} . Both \underline{a} and \bar{a} are isotone functions with respect to each individual parameter α_i and β_{ij} .*

Note that if all players enjoy the same direct benefits from actions ($\alpha_i = \alpha_j, \forall i, j \in N$), so that the social network is the sole source of asymmetry, then at the minimum equilibrium each player takes an action equal to the common benefit parameter, which is the optimal action in the absence of any status concerns. In this case, the minimum equilibrium does not precipitate any status losses.

We next prove that, when initiated at extreme points of the range of best responses, the best response dynamics converges in finite time to the extremal equilibria.

Proposition 2. *The best response dynamics initiated at the action profile $(\alpha_i)_{i \in N}$ converges in finite time to \underline{a} , while the one initiated at $(\alpha_i + \sum_{j \in N} \beta_{ij})_{i \in N}$ converges in finite time to \bar{a} .*

The following example provides a simple illustration of the convergence to the maximum equilibrium. We will use the network structure and payoffs from this example to illustrate several subsequent results.

Example 1. Best Response Dynamics. Consider the undirected network g pictured in Figure 1 connecting the set of players $N = \{1, 2, \dots, 7\}$. The existence of a link between nodes i and j indicates that $g_{ij} = g_{ji} = 1$; if there is no link between i and j then $g_{ij} = g_{ji} = 0$. Suppose that players have aggregate status concerns, i.e., $\beta = g$. To highlight the effects of status seeking, we assume that actions have no direct benefits, i.e., $\alpha_i = 0$ for all i . Clearly, in the minimum equilibrium $a_i = 0$ for all players i . We show that in the maximum equilibrium players 1, 2, 3, and 4 take action 3, while players 5, 6, and 7 take action 1.

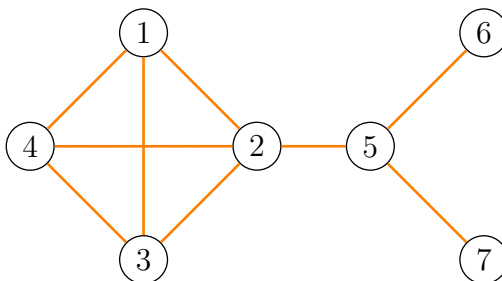


FIGURE 1. An undirected seven-player network for equilibrium examples

The convergence of the best response dynamics to the minimum equilibrium, starting from the profile in which every player i takes action $a_i = 0$, occurs immediately. To compute the maximum equilibrium, follow Proposition 2 and initiate the best response dynamics at $a = (\alpha_i + \sum_{j \in N} \beta_{ij})_{i \in N} = (3, 4, 3, 3, 3, 1, 1)$. Using Lemma 1, we find the next term in the best response dynamics, $B(a) = (3, 3, 3, 3, 1, 1, 1)$. At this action profile, player 2 drops his action from 4 to 3 since there are no status benefits from taking a higher action than all his neighbors. Similarly, player 5 drops his action from 3 to 1 because his marginal cost for keeping up with player 2 is 3, while his marginal status benefit is only 1. At the next iteration of the best response function, no player has incentives to further reduce his action, and we obtain $B^2(a) = (3, 3, 3, 3, 1, 1, 1) = B(a)$. By Proposition 2, the profile $(3, 3, 3, 3, 1, 1, 1)$ constitutes the maximum equilibrium, as claimed.

In Example 1, the minimum equilibrium obviously yields higher payoffs than the maximum equilibrium for all players. We next show that this conclusion extends to any pair of equilibria that can be ranked according to \succeq : that is, equilibria with higher actions are Pareto dominated. This result might appear straightforward, since each player i 's standard economic benefits are decreasing in a_i for $a_i \geq \alpha_i$. However, the ranking of players' actions can change and the magnitude of status losses can decrease as we move from lower to higher equilibria. In a strategic setting, though, other players respond, and player i 's cost increase (along with the negative externalities imposed by other players' high actions) exceeds the potential gains in status at higher equilibria. This finding captures, in a network setting, the results of Frank's (1985a) original formulation: players run ahead to stay in place.

Proposition 3. *For any ranked pair of equilibria of the status game $a \succeq a'$, each player has (weakly) higher utility at a' than at a , i.e., $u(a') \succeq u(a)$.*

The final result of this section shows that any increase in the status concerns β generates higher equilibrium activity and lower welfare. An increase in β_{ij} simply reflects the fact that player i places additional emphasis on his status relative to player j ; all else equal, player i has more incentives to increase his action. Increases in status concerns could arise in

many ways. Suppose, for example, that status concerns originate from an unweighted social network g . With the aggregate status specification $\beta = g$, the addition of a directed link ij to g translates into an increase in β_{ij} from 0 to 1. If instead $\beta = kg$ for some intensity parameter $k > 0$, an increase in k would lead to greater weights for all links, corresponding to greater disutility from all social comparisons. As in Proposition 3, any status gains resulting from the increased equilibrium activity induced by greater status concerns is outweighed by the higher costs (and the greater weight on social comparisons).

Proposition 4. *Consider two status games with parameters (α, β) and (α, β') where $\beta_{ij} \geq \beta'_{ij}$ for all pairs $i \neq j \in N$. Let u and u' denote the corresponding payoff functions. Then, for any equilibrium a' of the game (α, β') , there exists an equilibrium a of the game (α, β) such that $a \succeq a'$. Conversely, for any equilibrium a of the game (α, β) there is an equilibrium a' of (α, β') such that $a \succeq a'$. For any ranked equilibria $a \succeq a'$ of the respective games, it must be that $u'(a') \succeq u(a)$. In particular, for the minimum and maximum equilibria in the respective games, (\underline{a}, \bar{a}) and $(\underline{a}', \bar{a}')$, we have $u'(\underline{a}') \succeq u(\underline{a})$ and $u'(\bar{a}') \succeq u(\bar{a})$.*

Proposition 4 offers us guidance on when we might expect an expansion in the set of social connections in the underlying unweighted network g to generate larger equilibrium losses from status seeking. In a simple setting where the weight of existing links is not affected by the addition of new links (as in the case of aggregate status concerns), enlarging the set of social connections will lead to higher equilibrium actions. However, one can imagine situations in which additional links reduce the weights of existing links, as in the case of normalized status concerns derived from g . Then increased connectivity in the network g does not change all the entries in the matrix of status parameters β in the same direction. Thus, the particular specification of status concerns β as a function of the social network g determines the network comparative statics. We revisit the possibility that, under normalized status concerns, the addition of new links may decrease actions and increase payoffs in the maximum equilibrium in Example 5 from Section IV.

It should be emphasized that, as explained in Section A, the welfare comparative statics provided by Propositions 3 and 4 do not generalize to the version of the model in which actions have payoff externalities different from those stemming from social status comparisons.

III. COHESION AND EQUILIBRIUM SOCIAL CLASSES

In this section we study how the status concerns derived from the social network shape equilibrium outcomes. We first develop a notion of interconnectedness of sets of players, which we call *cohesion*. In more cohesive sets, players compare themselves more to one another, amplifying the returns of higher actions for social status. Second, we show that equilibria exhibit a stratification of the players into *status classes*. All players in the same

class take an identical action and the common action within each class is characterized by bounds that involve the cohesion of certain sets of players.

A. Cohesion. We define cohesion as a measure of the intensity of incentives and social comparisons within a set of players. For any set $S \subseteq N$ and each player $i \in S$, consider the standard economic benefits and the status gains obtained in social comparisons with other members of S resulting from an increase in a_i under the assumption that i is the lowest ranked player in S . For a marginal increase in a_i , player i experiences a marginal direct benefit of α_i and a marginal reduction in status loss of $\sum_{j \in S} \beta_{ij}$. The *cohesion* of S is defined by the lowest of these marginal returns to increasing a_i among $i \in S$.

Definition 1. The *cohesion*⁹ of a set of players $S \subseteq N$ is

$$c(S) = \min_{i \in S} \left(\alpha_i + \sum_{j \in S} \beta_{ij} \right).$$

Using the convention that $\min \emptyset = \infty$, we set $c(\emptyset) = \infty$.

In light of the discussion above, the cohesion of a set S can be interpreted as the highest common action that players in S have incentives to take in the absence of any status concerns relative to players outside S (under the assumption that $c(S) > \max_{i \in S} \alpha_i$; see Theorem 1 below). A set is more cohesive when its members' social comparisons are concentrated more within the set. When actions have no direct benefits and players have aggregate status concerns, the cohesion of a set of players is simply given by the minimum degree in the subgraph induced by those players, as illustrated in the following example.

Example 2. Cohesion of Sets. Consider the status game (α, β) derived from the unweighted network in Figure 1 as in Example 1. We have $c(\{1, 2, 3, 4\}) = 3$ since each player in the set $\{1, 2, 3, 4\}$ has exactly three neighbors within the set. When player 5 is added to the set $\{1, 2, 3, 4\}$, its cohesion drops from 3 to 1. Indeed, $c(\{1, 2, 3, 4, 5\}) = 1$ because player 5 has a single neighbor in the set $\{1, 2, 3, 4\}$. We can similarly compute $c(\{1, 2, 5\}) = 1$ and $c(\{1, 2, 6\}) = 0$.

Note that the cohesion function satisfies the following inequality,

$$(III.1) \quad c(S \cup S') \geq \min(c(S), c(S')), \forall S, S' \subseteq N,$$

which we exploit in our proofs.

⁹For the version of the model in which player i 's cost for action a_i is $f_i(a_i)$ discussed in Section A, the formula for cohesion should be modified as follows:

$$c(S) = \min_{i \in S} f_i'^{-1} \left(\alpha_i + \sum_{j \in S} \beta_{ij} \right).$$

B. Status Classes. It is useful to divide players into tiers according to the magnitude of their actions. We show that this tier partition captures the relevant equilibrium incentives: in equilibrium no player has incentives to move below or above her tier, and these incentives depend on the cohesion of sets related to higher tiers.

For any action profile a , we divide the players into tiers as follows. Let $\bar{k}(a)$ be the number of distinct actions players take at a and $\{a^1, a^2, \dots, a^{\bar{k}(a)}\} = \{a_i | i \in N\}$ denote the set of distinct action levels under a , in decreasing order: $a^1 > a^2 > \dots > a^{\bar{k}(a)}$. Denote by $C_k(a)$ the set of players taking the k^{th} highest action level under a , i.e., $C_k(a) = \{i \in N | a_i = a^k\}$. We refer to the set $C_k(a)$ as *status class k* under a and to its members as *status k players*.

The partition of players into status classes allows us to identify players' marginal status losses at an action profile a . A member i of status class k suffers status losses when comparing her action to those of neighbors in higher classes. Let

$$S_k(a) = \cup_{h=1}^k C_h(a)$$

be the set of players who take one of the k highest distinct actions. The set of players who take strictly higher actions than i at a is then given by $S_{k-1}(a)$.¹⁰ If i contemplates reducing her action at a by a small amount, she would suffer status losses with respect to her neighbors in $S_k(a)$.

The next result provides a necessary and sufficient condition for an action profile a to form an equilibrium in terms of the action levels a^k and the cohesion of the sets $S_k(a)$ and $S_{k-1}(a) \cup \{i\}$ with $i \in C_k(a)$. Consider a player i in status class k . In equilibrium, i cannot gain by increasing his action and reducing his status loss vis a vis higher class neighbors, who form the set $S_{k-1}(a)$. Nor can i gain by decreasing his action and reducing costs, albeit suffering further status losses with respect to higher class neighbors as well as his class k peers, who jointly form the group $S_k(a)$. Since cohesion captures the intensity of incentives as discussed above, the upper and lower bounds on the equilibrium actions a^k are derived from the cohesion of the relevant sets of players.

Theorem 1. *An action profile a is an equilibrium of the status game if and only if*

$$(III.2) \quad c(S_{k-1}(a) \cup \{i\}) \leq a^k \leq c(S_k(a)), \quad \forall i \in C_k(a), \forall k = 1, \dots, \bar{k}(a).$$

The following example illustrates the equilibrium characterization provided by Theorem 1 in the context of the network from Figure 1.

Example 3. Cohesion and Equilibrium Conditions. Consider again the network from Figure 1 with the underlying α and β specified as in Example 1. Suppose that a constitutes an equilibrium of the status game (α, β) formed by two social classes, $C_1(a) = \{1, 2, 3, 4\}$ and $C_2(a) = \{5, 6, 7\}$, which take actions a^1 and a^2 , respectively ($a^1 > a^2$). As discussed in

¹⁰It is notationally convenient to set $S_0(a) = \emptyset$.

Example 1, the maximum equilibrium has this class structure. The equilibrium conditions from Theorem 1 reduce to

$$c(\{i\}) \leq a^1 \leq c(\{1, 2, 3, 4\})$$

for each $i \in C_1(a)$ and

$$c(\{1, 2, 3, 4\} \cup \{i\}) \leq a^2 \leq c(\{1, 2, 3, 4, 5, 6, 7\})$$

for each $i \in C_2(a)$. The conditions $a^1 \leq c(\{1, 2, 3, 4\}) = 3$ and $a^2 \leq c(\{1, 2, 3, 4, 5, 6, 7\}) = 1$ essentially require that no player in each of the two classes has incentives to fall behind her class.

To identify the highest necessary lower bound on a^2 , recall that Example 2 showed that $c(\{1, 2, 3, 4, 5\}) = 1$. Since players 6 and 7 are not linked to any players in the first class, we have $c(\{1, 2, 3, 4, 6\}) = c(\{1, 2, 3, 4, 7\}) = 0$. Hence the lower bounds on a^2 boil down to the constraint $c(\{1, 2, 3, 4, 5\}) = 1 \leq a^2$, which guarantees that player 5 does not have incentives to increase her action and decrease her status loss vis a vis her only first class neighbor, player 2. Combining the conditions above, we find that in any equilibrium with the posited two-class structure, the low class must take action $a^2 = 1$ and the high class an action $a^1 \in (1, 3]$. Section V below explores the existence of multiple class equilibria in general networks.

IV. MAXIMUM EQUILIBRIUM AND COHESION

This section investigates the structure of the maximum equilibrium \bar{a} , which exhibits the most extreme escalation of actions due to status concerns. We find that the class stratification in the maximum equilibrium is driven by maximally cohesive sets. We first provide a top-down characterization of the equilibrium showing that the largest maximally cohesive subset of players forms the highest status class. High class players take actions equal to the cohesion of their class. Players in each successive class, who suffer status losses relative to higher classes, are similarly identified. We then develop a bottom-up characterization of the maximum equilibrium based on an algorithm that first determines the lowest class, then the second-lowest class, and so on. This alternative characterization delivers a method to compute the maximum equilibrium in polynomial time. At the end of the section, we use the characterizations of the maximum equilibrium to illuminate comparative statics with respect to the network of social connections and the corresponding specification of status concerns.

A. Top-Down Characterization of the Maximum Equilibrium. Here we construct a top-down algorithm that identifies the maximum equilibrium. By Theorem 1, no action in any equilibrium exceeds $\max_{M \subseteq N, M \neq \emptyset} c(M)$. We prove that this upper bound is achieved in the maximum equilibrium for some players. Once the highest status class in the maximum equilibrium is identified, we can use a similar idea to find the next highest status class, and

so on. The algorithm relies on the following recursive definition. Set $M_0 = \emptyset$. For $k \geq 1$, define M_k by¹¹

$$M_k = \bigcup_{\substack{M \in \arg \max \\ M' \supset M_{k-1}} c(M')} M.$$

The construction continues as long as $M_k \neq N$. Let \bar{k} denote the last step ($M_{\bar{k}} = N$).

Using (III.1), one can easily show that $c(M_k) = \max_{M \supset M_{k-1}} c(M)$. Thus M_k represents the largest (with respect to inclusion) maximally cohesive strict superset of M_{k-1} . It must be that $c(M_{k-1}) > c(M_k)$ for $k = 1, \dots, \bar{k}$.¹²

The result below directly relates the social classes and actions in the maximum equilibrium to the sets M_k and their levels of cohesion. The members of the largest maximally cohesive set M_1 drive one another to take the highest action at the maximum equilibrium, which is equal to the cohesion of this set. Since all players look up to the high status class M_1 , the second highest status class is formed by the remaining players who belong to the largest maximally cohesive superset of M_1 , and so forth.

Theorem 2. *The maximum equilibrium of the status game is characterized by*

$$\bar{a}_i = c(M_k), \forall i \in M_k \setminus M_{k-1}, k = 1, \dots, \bar{k}.$$

In other words, the maximum equilibrium exhibits exactly \bar{k} status classes, with class k given by $C_k(\bar{a}) = M_k \setminus M_{k-1}$ and its common action specified by $\bar{a}^k = c(M_k)$. The next result establishes that the action of each player in the maximum equilibrium is equal to the greatest cohesion of any set that includes the player.

Theorem 3. *The maximum equilibrium \bar{a} of the status game is given by*

$$\bar{a}_i = \max_{M \ni i} c(M), \forall i \in N.$$

B. The Bottom-Up Characterization of the Maximum Equilibrium. We now develop a procedure that first determines the lowest status class in the maximum equilibrium ($N \setminus M_{\bar{k}-1}$), then the second lowest class ($M_{\bar{k}-1} \setminus M_{\bar{k}-2}$), and so on. Let \mathcal{L} be the operator over subsets of N defined by

$$\mathcal{L}(M) = \bigcup_{M' \subset M, c(M') > c(M)} M'.$$

By (III.1), $c(\mathcal{L}(M)) > c(M)$. It follows that for every set M , $\mathcal{L}(M)$ is the largest subset of M (with respect to inclusion) that is strictly more cohesive than M . The definition implies that if M has no subsets that are more cohesive than itself, then $\mathcal{L}(M) = \emptyset$.

¹¹We use \subset and \supset to represent strict inclusions.

¹²For, $c(M_k) \geq c(M_{k-1})$ implies that $k \geq 2$ and $M_k \in \arg \max_{M \supset M_{k-2}} c(M)$, which means that $M_k \subseteq M_{k-1}$, a contradiction.

Note that $\mathcal{L}(N)$ is the union of all sets whose cohesion exceeds $c(N)$, which we can think of as sets of players who can push one another to take actions higher than $c(N)$. This intuition, along with the finding that $c(N)$ is the lowest action in the maximum equilibrium (Theorem 2), suggests that players outside $\mathcal{L}(N)$ must take action $c(N)$ and form the lowest class in \bar{a} . Once we pin down the lowest status class $C_{\bar{k}}(\bar{a})$, the remaining players suffer no status loss from social comparisons to $C_{\bar{k}}(\bar{a})$, which allows us to treat the strategic interaction among the players in $N \setminus C_{\bar{k}}(\bar{a})$ as an independent status game. Therefore, the lowest class in this reduced game constitutes the second to the lowest class in the original game, so we can successively identify the status classes in the maximum equilibrium proceeding from the bottom of the social hierarchy. Using this idea, the next result shows that the sequence $(M_k)_{k=0, \dots, \bar{k}}$ characterizing the maximum equilibrium can be alternatively derived by iterating the operator \mathcal{L} as follows. Find the smallest \bar{k} such that $\mathcal{L}^{\bar{k}}(N) = \emptyset$ ($\bar{k} \leq |N|$ since $\mathcal{L}(M) \subset M$ for all M), and then set $M_k = \mathcal{L}^{\bar{k}-k}(N)$ for all $k = 0, \dots, \bar{k}$.

Theorem 4. *The number of status classes \bar{k} in the maximum equilibrium is the smallest $k \geq 0$ such that $\mathcal{L}^k(N) = \emptyset$. Moreover, the sets characterizing the maximum equilibrium are given by $M_k = \mathcal{L}^{\bar{k}-k}(N)$ for all $k = 0, \dots, \bar{k}$.*

Theorem 4 offers a bottom-up perspective on the status stratification in the maximum equilibrium: the set $N \setminus \mathcal{L}(N)$ constitutes the lowest status class, the set $\mathcal{L}(N) \setminus \mathcal{L}^2(N)$ forms the second lowest class, and so forth.

C. Polynomial Time Computation of the Maximum Equilibrium. Both the top-down and the bottom-up computations of the maximum equilibrium entail the inspection of a number of subsets of nodes that grows exponentially in the number of players. We next develop a procedure that determines $\mathcal{L}(M)$ in polynomial time with respect to the cardinality of M for every $M \subseteq N$. In light of Theorems 2 and 4, this procedure renders an algorithm that has the methodological advantage of computing the maximum equilibrium using a number of basic operations that is a polynomial function of $|N|$.

Fix $M \subseteq N$. Let $L_0(M) = M$ and define a sequence $(L_s(M))$ recursively as follows. For $s \geq 1$, $L_s(M)$ denotes the subset of $L_{s-1}(M)$ obtained by simultaneously removing all nodes, if any, that “hold down” the cohesion of $L_{s-1}(M)$ to $c(M)$ or below. Formally,

$$L_s(M) = \left\{ i \in L_{s-1}(M) \mid \alpha_i + \sum_{j \in L_{s-1}(M)} \beta_{ij} > c(M) \right\}.$$

The construction ends at the first step s at which no new players are dropped from $L_{s-1}(M)$, i.e., $L_s(M) = L_{s-1}(M)$ (it is possible that the final set is empty). Denote this step by $\bar{s}(M)$. Clearly, $\bar{s}(M) \leq |M| + 1$. We prove that the final outcome of the procedure is $\mathcal{L}(M)$, i.e., $\mathcal{L}(M) = L_{\bar{s}(M)}(M)$.

Proposition 5. *For every set $M \subseteq N$, we have $\mathcal{L}(M) = L_{\bar{s}(M)}(M)$. In particular, $\mathcal{L}(M)$ can be computed in polynomial time with respect to $|M|$.*

Proposition 5 implies that players who “hold down” the cohesion of N to $c(N)$ belong to the lowest status class, as do players who keep the cohesion of the remaining subset at or below $c(N)$. The identification of low class players continues as long as the cohesion of the remaining players does not exceed $c(N)$. The next example illustrates the bottom-up algorithm, again using the 7-player unweighted network with aggregate status concerns from Figure 1.

Example 4. Bottom-Up Algorithm for Maximum Equilibrium. Consider the network from Figure 1 with α and β derived as in Example 1. We first need to compute $\mathcal{L}(N)$. Since $c(N) = 1$, we have $L_1(N) = \{1, 2, 3, 4, 5\}$, $L_2(N) = \{1, 2, 3, 4\} = L_3(N)$. Proposition 5 implies that $\mathcal{L}(N) = \{1, 2, 3, 4\}$. In order to evaluate $\mathcal{L}^2(N) = \mathcal{L}(\{1, 2, 3, 4\})$, we determine that $c(\{1, 2, 3, 4\}) = 3$ and then find $L_1(\{1, 2, 3, 4\}) = \emptyset = L_2(\{1, 2, 3, 4\})$. Proposition 5 yields $\mathcal{L}(\{1, 2, 3, 4\}) = \emptyset$. Theorem 4 implies that $\bar{k} = 2$, $M_1 = \{1, 2, 3, 4\}$, and $M_2 = N$. Then Theorem 2 delivers the maximum equilibrium: $\bar{a}_i = 3$ for $i \in \{1, 2, 3, 4\}$ and $\bar{a}_i = 1$ for $i \in \{5, 6, 7\}$.

D. Network Comparative Statics, Cohesion, and the Maximum Equilibrium. In this section we continue our discussion of the impact of expanding social ties and reference groups on equilibrium outcomes. The tools developed above allow us to trace how additional links and their effects on status concerns and cohesion affect the maximum equilibrium and thus the highest possible welfare losses from seeking social status. In Section B, we used Proposition 4 to argue that if social concerns are derived from unweighted networks using the aggregate specification, then the addition of new links increases actions and decreases welfare in the maximum equilibrium. Here we show that this conclusion does not extend to normalized status concerns. As the next example illustrates, under normalized status concerns, the addition of a link may dilute the cohesion of certain sets of players and lower the actions in the maximum equilibrium.

Example 5. New Links and the Maximum Equilibrium. We explore the effects of adding a directed link from player 2 to player 6 in the network from Figure 1, which leads to the network depicted in Figure 2. In both networks, we assume that actions have no intrinsic benefits ($\alpha_i = 0$ for all $i \in N$) and that status concerns are normalized. Recall that normalized status concerns are derived from network g according to the formula $\beta_{ij} = g_{ij}/(|N_i| + 1)$ for all $i, j \in N$. We use Theorems 2 and 4, along with Proposition 5, to find the maximum equilibrium for either network.

In the network from Figure 1, we find that $c(N) = 1/2$, and then compute $L_1(N) = \{1, 2, 3, 4, 5\}$, $L_2(N) = \{1, 2, 3, 4\} = L_3(N)$. By Proposition 5, $\mathcal{L}(N) = \{1, 2, 3, 4\}$. To

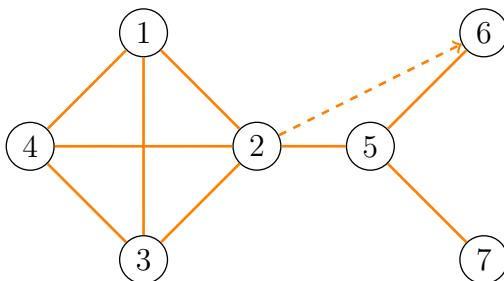


FIGURE 2. The addition of the link $(2, 6)$ decreases actions and increases payoffs in the maximum equilibrium.

evaluate $\mathcal{L}^2(N) = \mathcal{L}(\{1, 2, 3, 4\})$, we determine that $c(\{1, 2, 3, 4\}) = 3/5$ and then compute $L_1(\{1, 2, 3, 4\}) = \{1, 3, 4\}$, $L_2(\{1, 2, 3, 4\}) = \emptyset = L_3(\{1, 2, 3, 4\})$. Proposition 5 yields $\mathcal{L}(\{1, 2, 3, 4\}) = \emptyset$. Theorem 4 implies that $\bar{k} = 2$, $M_1 = \{1, 2, 3, 4\}$, and $M_2 = N$. Then Theorem 2 delivers the maximum equilibrium, $\bar{a}_i = 3/5$ for $i = 1, 2, 3, 4$ and $\bar{a}_i = 1/2$ for $i = 5, 6, 7$.

For the network depicted in Figure 2, we find that $c(N) = 1/2$, and the algorithm supporting Proposition 5 produces the following sequence:

$$L_1(N) = \{1, 2, 3, 4, 5\}, L_2(N) = \{1, 2, 3, 4\}, L_3(N) = \{1, 3, 4\}, L_4(N) = \emptyset = L_5(N).$$

Hence $\mathcal{L}(N) = \emptyset$, which, along with Theorems 2 and 4, implies that $M_1 = N$ and $\bar{a}_i = 1/2$ for all $i \in N$.¹³

Compared to the maximum equilibrium in the original network, the actions of players 1, 2, 3, and 4 decline from $3/5$ to $1/2$, while the actions of other players remain unchanged. Clearly, the addition of the link $(2, 6)$ increases every player's welfare under normalized status concerns, as the intrinsic payoffs of players 1, 2, 3, and 4 increase from the reduction in actions, while those of the other players do not change, and no player suffers status losses because a single status class emerges in the new maximum equilibrium.

This analysis demonstrates that the intuition of the corollary of Proposition 4, whereby the addition of new links that do not affect the status weights for existing links increases equilibrium activity and decreases welfare, does not carry over to the case of normalized status concerns. Indeed, with normalized status losses, the addition of the link $(2, 6)$ from the high status player 2 to the low status player 6 (status is evaluated at the maximum equilibrium for the original network) alleviates the status concerns of player 2 (by decreasing β_{2j} for each initial neighbor $j = 1, 3, 4$) and induces him to reduce his action. This, in turn, prompts players 1, 3, and 4 to lower their actions at the maximum equilibrium in the new network.

¹³In this specific example, the maximum equilibrium does not change if the link $(6, 2)$ is added to maintain an undirected network.

Even though the addition of links may decrease actions at the maximum equilibrium, it is worth noting that the empty network (with $g_{ij} = 0$ for all $i, j \in N$) generates the lowest equilibrium activity. Indeed, by Lemma 1, the unique equilibrium in the empty network—given by $(\alpha_i)_{i \in N}$ —is dominated in the order \succeq by every equilibrium for any other network with normalized status concerns. Moreover, if players have a common benefit parameter z and normalized status concerns, the complete network (defined by $g_{ij} = 1$ for all $i \neq j \in N$) supports the highest equilibrium activity among all networks.¹⁴ In this case, each player takes action $z + (|N| - 1)/|N|$ in the maximum equilibrium for the complete network. Since the cohesion of every non-empty set in any network with normalized status concerns cannot exceed $z + (|N| - 1)/|N|$, equilibrium actions in every network are bounded above by $z + (|N| - 1)/|N|$.

V. MULTIPLE-CLASS EQUILIBRIA

Finally, we investigate what networks of status concerns support equilibria in which players are divided into multiple social classes. We say that a pure strategy profile is a *class equilibrium* if it is an equilibrium of the status game that does not prescribe the same action for all players, so that multiple status classes emerge. Note that the maximum equilibrium exhibits multiple social classes if and only if there exists a nonempty subset of nodes that is more cohesive than N . The next example demonstrates that this condition is not necessary for the existence of class equilibria: in some networks, it is possible that \bar{a} is not a class equilibrium, yet class equilibria exist.

Example 6. Consider the undirected network depicted in Figure 3. Suppose that players do not gain direct benefits from their actions ($\alpha_i = 0$ for all $i \in N$) and have aggregate status concerns. In this case the maximum equilibrium \bar{a} gives rise to a single status class. Indeed, the characterizations of the maximum equilibrium from the previous section imply that $\bar{a}_i = 2$ for all players. However, by Theorem 1, a strategy profile a specified by $a_i = x$ for $i \in \{1, 2, 3\}$ and $a_i = y$ for $i \in \{4, 5, 6\}$ constitutes a class equilibrium whenever $x \neq y \in [1, 2]$.

The following result establishes that a class equilibrium exists if and only if we can find a nonempty set of nodes $M \neq N$ whose cohesion decreases when any single node is added to it. The intuition is that if M satisfies this property then the players in M may act as exclusive members of the highest status class in some equilibrium. The condition $c(M) > c(M \cup \{i\})$ for $i \in N \setminus M$ guarantees that player i does not have incentives to emulate the members of

¹⁴The following three-player example demonstrates that the assumption of a common benefit parameter is needed for this conclusion. Suppose that $N = \{1, 2, 3\}$, $\alpha_1 = 0$, $\alpha_2 = \alpha_3 = 1$. Then under normalized status concerns, the maximum equilibrium in the complete network is given by $a_1 = 2/3$, $a_2 = a_3 = 1 + 1/3$. The maximum equilibrium actions of players 2 and 3 increase to $1 + 1/2$ when they no longer have links to player 1 (the action of player 1 remains the same if he is still linked to 2 and 3).

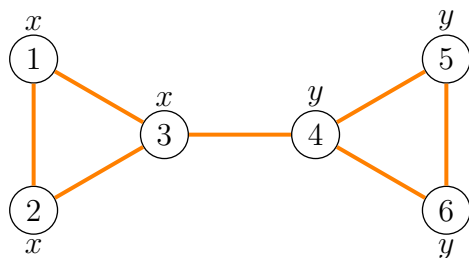


FIGURE 3. A collection of class equilibria for $x \neq y \in [1, 2]$

M if M forms the highest status class and its members take action $c(M)$. For instance, in Example 6 the set $M = \{4, 5, 6\}$ satisfies the condition as $2 = c(M) > c(M \cup \{i\}) \in \{0, 1\}$ for all $i \in N \setminus M$.

Theorem 5. *A class equilibrium exists if and only if there exists a nonempty set $M \subset N$ such that $c(M) > c(M \cup \{i\})$ for all $i \in N \setminus M$.*

VI. CONCLUSION

This paper studies social comparisons and status seeking activity. Our model captures two stylized facts: (1) reference groups for status comparisons are local, as described by a social network; and (2) people have aspirational status concerns and focus on upward social comparisons. These assumptions are grounded in the large economics and sociology literature on status and conspicuous consumption as well as many empirical studies on happiness and subjective well-being. As in previous theoretical work, in our model higher equilibrium levels of status seeking activity yield lower overall utility in a decentralized sort of rat race.

The analysis shows how the global network structure shapes the formation of social strata. The main finding is that the cohesion of sets of players determines the amount of status seeking activity. We prove that every equilibrium is characterized by a partition of the players into social classes, with actions in each class being constrained by the cohesion of certain sets of players.

Since equilibria that involve higher actions are Pareto dominated, our main analysis naturally focuses on the maximum equilibrium, which showcases the extreme inefficiencies created by status concerns. We provide a top-down characterization of the maximum equilibrium which iterates the finding that players in the most cohesive set(s) achieve the highest status. An alternative, bottom-up, characterization of this equilibrium builds on the observation that players who do not belong to any set that is more cohesive than the entire set of nodes form the lowest status class. The two characterizations reveal the rich combinatorial structure of the underlying network decomposition into status classes. Furthermore, the bottom-up characterization paves the way to a polynomial time algorithm for computing the maximum equilibrium.

We also show how changes in payoff parameters and network structure affect equilibria. When players are more concerned about status, they take higher actions and experience lower utility. This phenomenon is another version of the rat race outcome. The addition of new links to the network can have ambiguous effects on equilibrium actions and welfare. A new link increases the number of social comparisons. However, the direction in which actions at the maximum equilibrium shift depends on how status comparison weights relate to the network of social ties. With normalized status concerns, new links can decrease the cohesion of some sets and reduce status seeking activity. Finally, we identify a necessary and sufficient condition, which also relies on relative cohesion of various subsets of nodes, for the existence of multiple-class equilibria.

Future research in this area could take two related tracks. First, while the network of status concerns is fixed in our model, people can, at least partially, choose their social connections and reference groups.¹⁵ In our simple model, there are only losses from being linked to others. But of course people also benefit—emotionally and economically—from friendships and social interactions. A richer model of social status would involve network formation with benefits from friendships as well as status concerns. Second, a third party could be interested in designing a network in order to influence status seeking activity. Actions in our model can represent, for example, charitable contributions. People gain utility from contributing *per se* and reduce status loss by being among the top contributors.¹⁶ A fundraiser would then want to organize his or her events and campaign to highlight these comparisons to maximize overall contribution levels.

¹⁵Frank (1985a) is a well-known study of this phenomenon.

¹⁶Andreoni (1990) is a classic paper introducing the “warm glow” people experience from making charitable contributions. As for social comparisons, experiments show that people make greater contributions when they know others contribute larger amounts (Frey and Meier (2004) and Shang and Croson (2009)). Kumru and Vesterlund (2010) find that contributions by higher status subjects (where status is induced in the laboratory independently of contribution levels) influence the giving of other subjects.

APPENDIX A. PROOFS

Proof of Lemma 1. Fix $i \in N$ and a pure action profile a_{-i} for i 's opponents. The function $u_i(\cdot, a_{-i})$ is continuous, strictly concave, and semi-differentiable on the domain $[0, \infty)$. The left- and right-derivatives can be immediately computed,

$$(A.1) \quad \frac{\partial u_i(a_i-, a_{-i})}{\partial a_i} = \alpha_i - a_i + \sum_{\{j \in N | a_j \geq a_i\}} \beta_{ij}$$

$$(A.2) \quad \frac{\partial u_i(a_i+, a_{-i})}{\partial a_i} = \alpha_i - a_i + \sum_{\{j \in N | a_j > a_i\}} \beta_{ij} = \alpha_i - a_i + r_i(a_i, a_{-i}).$$

Since $\lim_{a_i \rightarrow \infty} u_i(a_i, a_{-i}) = -\infty$, $u_i(\cdot, a_{-i})$ admits a unique maximizer a_i^* in the interval $[0, \infty)$.

Since a_i^* is the maximizer of $u_i(\cdot, a_{-i})$ on $[0, \infty)$, it must be that $\partial u_i(a_i^+, a_{-i})/\partial a_i \leq 0$. Along with (A.2), the last inequality implies that

$$(A.3) \quad a_i^* \geq \alpha_i + r_i(a_i^*, a_{-i}).$$

If $a_i^* = 0$, then clearly $a_i^* = \min \{a_i \mid a_i \geq \alpha_i + r_i(a_i, a_{-i})\}$. Suppose instead that $a_i^* > 0$. Then $\partial u_i(a_i^-, a_{-i})/\partial a_i \geq 0$ because a_i^* maximizes $u_i(\cdot, a_{-i})$ on $[0, \infty)$. Formula (A.1) leads to

$$a_i^* \leq \alpha_i + \sum_{\{j \in N | a_j \geq a_i^*\}} \beta_{ij}.$$

It follows that for $a_i < a_i^*$, we have

$$(A.4) \quad a_i < a_i^* \leq \alpha_i + \sum_{\{j \in N | a_j \geq a_i^*\}} \beta_{ij} \leq \alpha_i + \sum_{\{j \in N | a_j > a_i\}} \beta_{ij} = \alpha_i + r_i(a_i, a_{-i}).$$

The last inequality is a consequence of $\{j \in N | a_j \geq a_i^*\} \subseteq \{j \in N | a_j > a_i\}$ for $a_i < a_i^*$. From (A.3) and (A.4), we infer that $a_i^* = \min \{a_i \mid a_i \geq \alpha_i + r_i(a_i, a_{-i})\}$ (in particular, the minimum exists). Therefore, $a_i^* = B_i(a_{-i})$ is the unique best response of player i to a_{-i} . \square

Proof of Lemma 2. Fix an action profile a and let $(a^t)_{t \geq 0}$ denote the best response dynamics initiated at a . We prove by induction on t that

$$\forall i \in N, a_i^t \in A := \{a_j \mid j \in N\} \cup \left\{ \alpha_j + \sum_{l \in S} \beta_{jl} \mid j \in N, S \subseteq N \right\}$$

for all $t \geq 0$. The base case $t = 0$ is trivially verified ($a^0 = a$).

Assuming that the induction hypothesis holds for all lower values, we set out to prove it for $t \geq 1$. Suppose, by contradiction, there exists $i \in N$ such that $a_i^t \notin A$. By definition, $a_i^t = B_i(a_{-i}^{t-1})$. Lemma 1 leads to $a_i^t \geq \alpha_i + r_i(a_i^t, a_{-i}^{t-1})$. Since $a_i^t \notin A$ and $\alpha_i + r_i(a_i^t, a_{-i}^{t-1}) \in A$, it must be that $a_i^t > \alpha_i + r_i(a_i^t, a_{-i}^{t-1})$. Hence there exists $\varepsilon > 0$ such that

$$(A.5) \quad a_i^t - \varepsilon > \alpha_i + r_i(a_i^t, a_{-i}^{t-1}).$$

The induction hypothesis guarantees that $a_j^{t-1} \in A$ for all $j \in N$. Since $a_i^t \notin A$, it follows that $a_i^t \notin \{a_j^{t-1} \mid j \in N\}$. Then there exists $\varepsilon' \in (0, \varepsilon)$ such that

$$[a_i^t - \varepsilon', a_i^t] \cap \{a_j^{t-1} \mid j \in N\} = \emptyset.$$

It follows that $r_i(a_i^t - \varepsilon', a_{-i}^{t-1}) = r_i(a_i^t, a_{-i}^{t-1})$. Then the inequality (A.5) implies that

$$a_i^t - \varepsilon' > a_i^t - \varepsilon > \alpha_i + r_i(a_i^t, a_{-i}^{t-1}) = \alpha_i + r_i(a_i^t - \varepsilon', a_{-i}^{t-1}).$$

Along with Lemma 1, the condition $a_i^t - \varepsilon' > \alpha_i + r_i(a_i^t - \varepsilon', a_{-i}^{t-1})$ leads to $B_i(a_{-i}^{t-1}) \leq a_i^t - \varepsilon'$, which contradicts the assumption that $a_i^t = B_i(a_{-i}^{t-1})$. The contradiction completes the proof of the inductive step. Therefore, all the terms of the best response dynamics initiated at a belong to the finite set A^N . \square

Proof of Proposition 1. By Lemma 1, $B_i(a_{-i}) \leq \alpha_i + \sum_{j \in N} \beta_{ij}, \forall i, a_{-i}$. The game with each player i 's actions restricted to the compact set $[0, \alpha_i + \sum_{j \in N} \beta_{ij}]$ is supermodular (see the definition in Section 2 of Milgrom and Roberts (1990)). To establish this fact, it is sufficient to show that $-\max(a_j - a_i, 0)$ has increasing differences in (a_i, a_j) . Fix $a_j > a'_j \geq 0$. We have to prove that the expression $-\max(a_j - a_i, 0) + \max(a'_j - a_i, 0)$ is increasing in a_i . This follows immediately from noting that

$$-\max(a_j - a_i, 0) + \max(a'_j - a_i, 0) = \begin{cases} a'_j - a_j & \text{if } a_i \leq a'_j \\ a_i - a_j & \text{if } a_i \in (a'_j, a_j) \\ 0 & \text{if } a_i \geq a_j \end{cases}$$

is an increasing function in the variable a_i . Moreover, the payoff function u_i has increasing differences in (a_i, α_j) and (a_i, β_{jk}) for all $j, k \in N$. This follows from the general observation that if f is an increasing function of x (that does not depend on y) and g is an increasing function of y (that does not depend on x) then the product $f(x)g(y)$ has increasing differences in (x, y) . In light of these remarks, the proposition becomes a corollary of the results of Milgrom and Roberts (1990). \square

Proof of Proposition 2. Let a^0 be the action profile specified by $a_i^0 = \alpha_i$ for $i \in N$ and $(a^t)_{t \geq 0}$ denote the best response dynamics initiated at a^0 . Since \underline{a} is an equilibrium of the status game, we have $\underline{a} = B^t(\underline{a})$ for all $t \geq 0$. By Lemma 1, $\underline{a}_i = B_i(\underline{a}_{-i}) \geq \alpha_i = a_i^0$ for all $i \in N$, so $\underline{a} \succeq a^0$. As B is an isotone function, the relation $\underline{a} \succeq a^0$ leads to

$$(A.6) \quad \underline{a} = B^t(\underline{a}) \succeq B^t(a^0) = a^t, \forall t \geq 0.$$

By Lemma 1, $B_i(a_{-i}^0) \geq \alpha_i = a_i^0$ for all $i \in N$, so $B(a^0) \succeq a^0$. Corollary 1 then implies that the best response dynamics initiated at a^0 converges in finite time \bar{t} to an equilibrium $a^{\bar{t}}$. By Proposition 1,

$$(A.7) \quad a^{\bar{t}} \succeq \underline{a}.$$

Then (A.6) and (A.7) imply that $a^{\bar{t}} = \underline{a}$, which establishes the first part of the result. The second part is proven analogously. \square

Proof of Proposition 3. Fix a pair of equilibria $a \succeq a'$ and a player $i \in N$. The assumption that a' is an equilibrium implies that $u_i(a') \geq u_i(a_i, a'_{-i})$. Since u_i is decreasing in player j 's action and $a_j \geq a'_j$ for all $j \neq i$, we have $u_i(a_i, a'_{-i}) \geq u_i(a_i, a_{-i}) = u_i(a)$. Stringing together the inequalities above, we obtain that $u_i(a') \geq u_i(a)$. It follows that $u(a') \succeq u(a)$. \square

Proof of Proposition 4. Define r and r' by

$$\begin{aligned} r_i(a_i, a_{-i}) &= \sum_{\{j|a_j > a_i\}} \beta_{ij} \\ r'_i(a_i, a_{-i}) &= \sum_{\{j|a_j > a_i\}} \beta'_{ij}. \end{aligned}$$

Let B_i and B denote the best response functions in the game (α, β) , as defined in Section A. Suppose that a' is a Nash equilibrium in the game (α, β') . Fix $i \in N$. By Lemma 1, the fact that a'_i is a best response for player i to his opponents' action profile a'_{-i} in the game (α, β') implies that

$$(A.8) \quad \tilde{a}_i < \alpha_i + r'_i(\tilde{a}_i, a'_{-i}) \leq \alpha_i + r_i(\tilde{a}_i, a'_{-i}), \forall \tilde{a}_i \in [0, a'_i],$$

which combined with Lemma 1 leads to $B_i(a'_{-i}) \geq a'_i$. This establishes that $B(a') \succeq a'$. Corollary 1 then implies that the best response dynamics for the game (α, β) initiated at a' converges in finite time to an equilibrium $a \succeq a'$. The proof for the converse statement is analogous.

As to the welfare claim, consider an equilibrium a of the game (α, β) and an equilibrium a' of the game (α, β') such that $a \succeq a'$. Then the following string of inequalities holds

$$(A.9) \quad u'_i(a') = u'_i(a'_i, a'_{-i}) \geq u'_i(a_i, a'_{-i}) \geq u_i(a_i, a'_{-i}) \geq u_i(a_i, a_{-i}) = u_i(a), \forall i \in N.$$

The first inequality follows from the fact that a' is an equilibrium for the game with payoffs u' , the second one is a consequence of the assumption that $\beta_{ij} \geq \beta'_{ij}$ for $j \neq i$ and the observation that player i 's payoff is decreasing in each of his status parameters, and the last one relies on the hypothesis that $a_j \geq a'_j$ for all $j \neq i$ ($a \succeq a'$) and the fact that u_i is decreasing with respect to the action of every opponent j . Then (A.9) establishes that $u'(a') \succeq u(a)$. The last part of the proposition follows from the argument above, as $\underline{a} \succeq \underline{a}'$ and $\bar{a} \succeq \bar{a}'$ by Proposition 1. \square

Proof of Theorem 1. We first establish the ‘‘only if’’ part. Suppose that a is a Nash equilibrium and fix $k \in \overline{1, k(a)}$ and $i \in C_k(a)$. Then $B_i(a_{-i}) = a_i$ leads to

$$a_i \geq \alpha_i + r_i(a_i, a_{-i}) = \alpha_i + \sum_{j \in S_{k-1}(a)} \beta_{ij} \geq c(S_{k-1}(a) \cup \{i\}),$$

proving the lower bound.

For $j \in S_k(a)$, $B_j(a_{-j}) = a_j \geq a_i$ implies that for any $a'_j \in [a_h, a_i]$ with $h \in C_{k+1}(a)$ (or $a'_j \in [0, a_i]$ if $k = \bar{k}(a)$),

$$a'_j < \alpha_j + r_j(a'_j, a_{-j}) = \alpha_j + \sum_{l \in S_k(a)} \beta_{jl}.$$

Taking the limit $a'_j \uparrow a_i$, we obtain that

$$a_i \leq \alpha_j + \sum_{l \in S_k(a)} \beta_{jl}.$$

Since the inequality above holds for all $j \in S_k(a)$, it follows that

$$a_i \leq \min_{j \in S_k(a)} \left(\alpha_j + \sum_{l \in S_k(a)} \beta_{jl} \right) = c(S_k(a)),$$

which establishes the upper bound.

We next prove the “if” part. Suppose that a is an action profile satisfying (III.2) and fix $k \in \underline{1}, \bar{k}(a)$ and $i \in C_k(a)$. If $k \geq 2$, then for $j \in C_{k-1}(a)$, we have

$$c(S_{k-1}(a) \cup \{i\}) \leq a_i < a_j \leq c(S_{k-1}(a)).$$

In particular, it must be that $c(S_{k-1}(a) \cup \{i\}) < c(S_{k-1}(a))$, which means that

$$c(S_{k-1}(a) \cup \{i\}) = \alpha_i + \sum_{l \in S_{k-1}(a)} \beta_{il}.$$

Then we obtain that

$$a_i \geq \alpha_i + r_i(a_i, a_{-i}).$$

Note that the inequality $a_i \geq \alpha_i + r_i(a_i, a_{-i})$ is also satisfied if $k = 1$ since in that case $r_i(a_i, a_{-i}) = 0$, and (III.2) requires that $a_i \geq c(\{i\}) = \alpha_i$.

Moreover, for $a'_i < a_i$ we have

$$a'_i < a_i \leq c(S_k(a)) \leq \alpha_i + \sum_{l \in S_k(a)} \beta_{il} \leq \alpha_i + r_i(a'_i, a_{-i}).$$

The inequalities above establish that $B_i(a_{-i}) = a_i$. Since the last condition holds for all players i , the action profile a constitutes a Nash equilibrium. \square

Proof of Theorem 2. Set $\tilde{a}_i = c(M_k)$ for all $i \in M_k \setminus M_{k-1}$ and $k = 1, \dots, \bar{k}$. We first establish that $(\tilde{a}_i)_{i \in N}$ constitutes a Nash equilibrium of the status game and then prove that $\bar{a}_i = \tilde{a}_i$ for all $i \in N$.

By construction, for $i \in M_k \setminus M_{k-1}$, we have $c(M_{k-1} \cup \{i\}) \leq c(M_k)$. Hence

$$(A.10) \quad c(M_{k-1} \cup \{i\}) \leq \tilde{a}_i = c(M_k), \forall i \in M_k \setminus M_{k-1}, \forall k = 1, \dots, \bar{k}.$$

Since $c(M_{k-1}) > c(M_k)$ for $k = 1, \dots, \bar{k}$, we have $\bar{k}(\tilde{a}) = \bar{k}$ and $S_k(\tilde{a}) = M_k$ for $k = 1, \dots, \bar{k}$. Then (A.10) becomes

$$c(S_{k-1}(\tilde{a}) \cup \{i\}) \leq \tilde{a}_i = c(S_k(\tilde{a})), \forall i \in S_k(\tilde{a}) \setminus S_{k-1}(\tilde{a}), \forall k = 1, \dots, \bar{k}(\tilde{a}).$$

By Theorem 1, \tilde{a} constitutes an equilibrium. Proposition 1 implies that $\tilde{a}_i \leq \bar{a}_i$ for all $i \in N$.

To prove that $\bar{a}_i = \tilde{a}_i$ for all $i \in N$, we proceed by contradiction. Suppose that $\bar{a}_i > \tilde{a}_i$ for some $i \in M_k \setminus M_{k-1}$. Define $\tilde{M} = \{j \in N | \bar{a}_j > \tilde{a}_j\}$. Since $\bar{a}_j \geq \tilde{a}_j > \tilde{a}_i$ for all $j \in M_{k-1}$, it must be that $\tilde{M} \supseteq M_{k-1} \cup \{i\}$.

For all $j \in \tilde{M}$, $B_j(\bar{a}_{-j}) = \bar{a}_j > \tilde{a}_i$ implies that

$$\tilde{a}_i < \alpha_j + r_j(\tilde{a}_i, \bar{a}_{-j}) = \alpha_j + \sum_{l \in \tilde{M}} \beta_{jl}.$$

Then $\tilde{a}_i = c(M_k)$ leads to

$$c(M_k) < \alpha_j + \sum_{l \in \tilde{M}} \beta_{jl}.$$

Since the last inequality holds for all $j \in \tilde{M}$, we obtain that

$$c(M_k) < \min_{j \in \tilde{M}} \left(\alpha_j + \sum_{l \in \tilde{M}} \beta_{jl} \right) = c(\tilde{M}).$$

However, $\tilde{M} \supset M_{k-1}$ and $c(\tilde{M}) > c(M_k)$ contradict the fact that $c(M_k) = \max_{M \supset M_{k-1}} c(M)$. \square

Proof of Theorem 3. Consider a player $i \in M_k \setminus M_{k-1}$. By Theorem 2, we have $\bar{a}_i = c(M_k)$. Suppose, by contradiction, that there exists $M \ni i$ such that $c(M) > c(M_k)$. Then $c(M \cup M_{k-1}) \geq \min(c(M), c(M_{k-1}))$ (III.1), along with $c(M) > c(M_k)$ and $c(M_{k-1}) > c(M_k)$, implies that $c(M \cup M_{k-1}) > c(M_k)$. Since $i \in M_k \setminus M_{k-1}$ and $i \in M$, we have $M \cup M_{k-1} \supset M_{k-1}$. Then $c(M \cup M_{k-1}) > c(M_k)$ contradicts the fact that $c(M_k)$ represents the greatest cohesion achieved by any strict superset of M_{k-1} . \square

Proof of Theorem 4. We prove that the nested sequence of nodes characterizing \bar{a} satisfies $\mathcal{L}(M_k) = M_{k-1}$ for $k = 1, \dots, \bar{k}$. First, note that $M_{k-1} \subseteq \mathcal{L}(M_k)$ since $M_{k-1} \subset M_k$ and $c(M_{k-1}) > c(M_k)$. Moreover, the definition of \mathcal{L} , along with (III.1), implies that $c(\mathcal{L}(M_k)) > c(M_k)$. If $M_{k-1} \subset \mathcal{L}(M_k)$, the condition $c(\mathcal{L}(M_k)) > c(M_k)$ contradicts the fact that M_k achieves the highest cohesion among all strict supersets of M_{k-1} . Hence $\mathcal{L}(M_k) = M_{k-1}$.

The operator \mathcal{L} can be iterated to pin down the entire sequence (M_k) . Indeed, since $M_{\bar{k}} = N$ and $M_{k-1} = \mathcal{L}(M_k)$ for $k = 1, \dots, \bar{k}$, it must be that $M_k = \mathcal{L}^{\bar{k}-k}(N)$ for $k = 0, \dots, \bar{k}$. Then $M_0 = \emptyset$ and $M_1 \neq \emptyset$ imply that $\mathcal{L}^{\bar{k}}(N) = \emptyset$ and $\mathcal{L}^{\bar{k}-1}(N) \neq \emptyset$. Therefore, \bar{k} is the smallest $k \geq 0$ such that $\mathcal{L}^k(N) = \emptyset$. \square

Proof of Proposition 5. Using the facts that $\mathcal{L}(M) \subset M$ and $c(\mathcal{L}(M)) > c(M)$, we can show by induction on s that $\mathcal{L}(M) \subseteq L_s(M)$ for $s = 0, \dots, \bar{s}(M)$. In particular, $\mathcal{L}(M) \subseteq L_{\bar{s}(M)}(M)$. Since $L_{\bar{s}(M)}(M) = L_{\bar{s}(M)-1}(M)$, it must be that

$$\alpha_i + \sum_{j \in L_{\bar{s}(M)}(M)} \beta_{ij} > c(M)$$

for all $i \in L_{\bar{s}(M)}(M)$, which implies that $c(L_{\bar{s}(M)}(M)) > c(M)$. Then $L_{\bar{s}(M)}(M) \subseteq \mathcal{L}(M)$. Therefore, $\mathcal{L}(M) = L_{\bar{s}(M)}(M)$. Clearly, $L_{\bar{s}(M)}(M)$ can be computed in polynomial time with respect to $|M|$. \square

Proof of Theorem 5. We first establish the “only if” part. Suppose that a is a class equilibrium. Define $M = S_1(a)$ and $z = \max_{i \in N} a_i$. We have $M \subset N$ because a is a class equilibrium. Since all players in M take action z under a , Theorem 1 implies that

$$(A.11) \quad z \leq c(M).$$

Now fix $i \in N \setminus M$. Then $B_i(a_{-i}) = a_i < z$ implies that

$$(A.12) \quad z > a_i \geq \alpha_i + r_i(a_i, a_{-i}) \geq \alpha_i + \sum_{j \in M} \beta_{ij} \geq c(M \cup \{i\}).$$

Combining (A.11) and (A.12), we obtain that $c(M) > c(M \cup \{i\})$ for all $i \in N \setminus M$.

We next prove the “if” part. Let $M \subset N$ be a nonempty set such that $c(M) > c(M \cup \{i\})$ for all $i \in N \setminus M$. The definition of c implies that

$$c(M \cup \{i\}) = \alpha_i + \sum_{j \in M} \beta_{ij}, \forall i \in N \setminus M.$$

Let $x = \max_{i \in N \setminus M} c(M \cup \{i\})$. We have

$$(A.13) \quad c(M) > x \geq c(M \cup \{i\}), \forall i \in N \setminus M.$$

Consider the strategy profile a^0 defined by

$$\begin{aligned} a_i^0 &= c(M), \forall i \in M \\ a_i^0 &= x, \forall i \in N \setminus M. \end{aligned}$$

Let $(a^t)_{t \geq 0}$ denote the best response dynamics initiated at a^0 . We prove by induction on t that for all $t \geq 0$, $a_i^{t+1} \geq a_i^t$ if $i \in M$ and $a_i^{t+1} \leq a_i^t$ if $i \in N \setminus M$.

For the induction base case $t = 0$, we have to show that $a_i^1 = B_i(a_{-i}^0) \geq c(M) = a_i^0$ if $i \in M$ and $a_i^1 = B_i(a_{-i}^0) \leq x = a_i^0$ if $i \in N \setminus M$. To prove that $B_i(a_{-i}^0) \geq c(M)$ for $i \in M$, note that if $a_i < c(M)$ then

$$a_i < c(M) \leq \alpha_i + \sum_{j \in M} \beta_{ij} \leq \alpha_i + \sum_{j \in N: a_j^0 > a_i} \beta_{ij} = \alpha_i + r_i(a_i, a_{-i}^0).$$

Lemma 1 then implies that $B_i(a_{-i}^0) \geq c(M)$.

We next show that $B_i(a_{-i}^0) \leq x$ for $i \in N \setminus M$. For $i \in N \setminus M$, we have

$$x \geq c(M \cup \{i\}) = \alpha_i + \sum_{j \in M} \beta_{ij} = \alpha_i + \sum_{j \in N: a_j^0 > x} \beta_{ij} = \alpha_i + r_i(x, a_{-i}^0).$$

By Lemma 1, it must be that $B_i(a_{-i}^0) \leq x$.

Assuming that the inductive hypothesis holds for all smaller values, we now prove the inductive step for $t \geq 1$. We first show that if $i \in M$, then $a_i^{t+1} = B_i(a_{-i}^t) \geq a_i^t$. Fix $i \in M$ and $a_i < a_i^t$. Let $a'_i = \max(a_i, x)$. By the inductive hypothesis, we have $a_i^t \geq c(M) > x$, so $a'_i < a_i^t = B_i(a_{-i}^{t-1})$. Then Lemma 1 implies that $a'_i < \alpha_i + r_i(a'_i, a_{-i}^{t-1})$. Since $a_j^t \leq a_j^{t-1} \leq a_j^0 = x$ for $j \in N \setminus M$ by the inductive hypothesis and $a'_i \geq x$, we have

$$\begin{aligned} r_i(a'_i, a_{-i}^{t-1}) &= \sum_{j \in M: a_j^{t-1} > a'_i} \beta_{ij} \\ r_i(a'_i, a_{-i}^t) &= \sum_{j \in M: a_j^t > a'_i} \beta_{ij}. \end{aligned}$$

Also by the inductive hypothesis, we have $a_j^t \geq a_j^{t-1}$ for $j \in M$, which implies that $r_i(a'_i, a_{-i}^{t-1}) \leq r_i(a'_i, a_{-i}^t)$. It follows that

$$a_i \leq a'_i < \alpha_i + r_i(a'_i, a_{-i}^{t-1}) \leq \alpha_i + r_i(a'_i, a_{-i}^t) \leq \alpha_i + r_i(a_i, a_{-i}^t).$$

We established that $a_i < \alpha_i + r_i(a_i, a_{-i}^t)$ for every $a_i < a_i^t$, which along with Lemma 1 implies that $a_i^{t+1} = B_i(a_{-i}^t) \geq a_i^t$, as desired.

We next prove that $a_i^{t+1} = B_i(a_{-i}^t) \leq a_i^t$ for $i \in N \setminus M$. Fix $i \in N \setminus M$. By the inductive hypothesis, we have $a_j^t \geq a_j^{t-1} \geq c(M) > x \geq a_i^t$ for all $j \in M$. It follows that

$$\begin{aligned} r_i(a_i^t, a_{-i}^{t-1}) &= \sum_{j \in M: a_j^{t-1} > a_i^t} \beta_{ij} + \sum_{j \in N \setminus M: a_j^{t-1} > a_i^t} \beta_{ij} = \sum_{j \in M} \beta_{ij} + \sum_{j \in N \setminus M: a_j^{t-1} > a_i^t} \beta_{ij} \\ r_i(a_i^t, a_{-i}^t) &= \sum_{j \in M: a_j^t > a_i^t} \beta_{ij} + \sum_{j \in N \setminus M: a_j^t > a_i^t} \beta_{ij} = \sum_{j \in M} \beta_{ij} + \sum_{j \in N \setminus M: a_j^t > a_i^t} \beta_{ij}. \end{aligned}$$

Since $a_j^{t-1} \geq a_j^t$ for all $j \in N \setminus M$ by the inductive hypothesis, we obtain that

$$\sum_{j \in N \setminus M: a_j^{t-1} > a_i^t} \beta_{ij} \geq \sum_{j \in N \setminus M: a_j^t > a_i^t} \beta_{ij},$$

and hence $r_i(a_i^t, a_{-i}^{t-1}) \geq r_i(a_i^t, a_{-i}^t)$.

Note that the condition $a_i^t = B_i(a_{-i}^{t-1})$ and Lemma 1 lead to $a_i^t \geq \alpha_i + r_i(a_i^t, a_{-i}^{t-1})$. Therefore, $a_i^t \geq \alpha_i + r_i(a_i^t, a_{-i}^t)$, which along with Lemma 1 implies that $a_i^{t+1} = B_i(a_{-i}^t) \leq a_i^t$, as needed.

The boundedness and monotonicity of the sequence $(a_i^t)_{t \geq 0}$ for every $i \in N$, combined with Lemma 2, imply that $(a^t)_{t \geq 0}$ converges in a finite number of steps \bar{t} to an equilibrium $a^{\bar{t}}$. Furthermore, we showed that $a_i^t \geq a_i^0 = c(M) > x = a_j^0 \geq a_j^t$ for all $i \in M, j \in N \setminus M$

and $t = 0, \dots, \bar{t}$. Therefore, $a_i^{\bar{t}} > a_j^{\bar{t}}$ for all $i \in M, j \in N \setminus M$, so $a^{\bar{t}}$ constitutes a class equilibrium. \square

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